

Transversely Polarized Drell-Yan Process and Soft Gluon Resummation in QCD

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Abstract

We calculate the transverse-momentum Q_T spectrum of the dilepton in the transversely polarized Drell-Yan process on the basis of the factorization theorem in QCD. We take into account universal logarithmically enhanced corrections in edge region of phase space by resumming multiple soft-gluon emissions to all orders in the small Q_T region.

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§1. Introduction

The nucleon structure appearing in high energy processes has been studied for a long time on the basis of quantum chromodynamics (QCD). The basic theoretical framework has been developed as the “factorization theorem” in QCD.¹⁾ As a result of factorization, the physical quantities (cross sections) are obtained as a convolution of the short- and long-distance parts. The former contains all the dependence on the hard scale, and the latter is controlled by the nonperturbative dynamics of QCD. We can apply perturbation theory to the short-distance part, thanks to the asymptotically free nature of QCD. The long-distance parts, however, can be determined only by experiments or some nonperturbative method, such as lattice QCD. The importance and advantage of perturbative QCD based on the factorization theorem reside in the fact that this theorem allows us to define the long-distance parts as process-independent universal objects, which are represented unambiguously as nucleon matrix elements of the operators of quarks and/or gluons and are often much simpler than the original quantities.

For spin-independent processes, the many experiments performed to this time have helped us to determine the long-distance, nonperturbative parts as “parton distributions” inside the nucleon,²⁾ and we have obtained a consistent understanding of the perturbative and nonperturbative dynamics, except in “edge regions” of phase space.¹⁾ For spin-dependent processes, however, our understanding is still poor, and many questions remain unanswered.^{3),4)} Therefore, to understand spin-dependent processes and the spin structure of nucleons through them is an important problem. Furthermore, spin-dependent quantities are, in general, believed to be quite sensitive to the structure of interactions among particles. These are the reasons for the great amount of activity recently in high-energy spin physics.

It is expected that a number of ongoing polarization experiments, such as RHIC-Spin,^{5),6)} using polarized proton-proton collisions,⁷⁾ HERMES⁸⁾ and COMPASS,⁹⁾ using lepton scattering off polarized protons, etc., will provide important experimental data to reveal spin-dependent phenomena associated with the structure of nucleons. We also expect that data from future polarization experiments using proton-proton collisions at J-PARC,¹⁰⁾ proton-antiproton collisions at GSI,¹¹⁾ etc will provide useful information. Therefore, it is important and interesting to investigate various processes to be studied in those experiments.

Among the many spin-dependent processes, the polarized Drell-Yan (DY) process plays a unique role. Indeed, transversity^{12)–14)} is one of the characteristic observables which can be measured in DY using a transversely polarized beam. The transversity $\delta q(x)$ is a twist-2 parton distribution associated with the probability distribution of transversely polarized quarks inside transversely polarized nucleons, i.e., the partonic structure of nucleons which

is complementary to that represented by the other twist-2 distributions, such as the familiar density and helicity distributions $q(x)$ and $\Delta q(x)$. However, $\delta q(x)$ is not yet well understood, because $\delta q(x)$ cannot be measured in inclusive deep inelastic scattering (DIS) due to its “chiral-odd” nature.^{12)–14)} In this paper, we focus on the transversely polarized Drell-Yan (tDY) process, $p^\uparrow p^\uparrow \rightarrow l^+ l^- X$, producing the dilepton $l^+ l^-$ with an invariant mass $Q \gg \Lambda_{\text{QCD}}$ in the final state, as a process that is likely to allow investigation of the transversity $\delta q(x)$. Our aim is to calculate the QCD corrections to the tDY cross section on the basis of the QCD factorization framework, establishing control over the large higher-order corrections near edge region of phase space.

Simpleminded calculations of QCD corrections to the DY cross section suffer from ultraviolet (UV) and infrared (IR) divergences due to the loops associated with (massless) quarks and gluons. The UV divergence can be regularized and renormalized straightforwardly using a standard procedure, and therefore it does not pose any problem. By contrast, the IR divergence is more intricate and must be treated using the factorization theorem:¹⁾ Introducing an appropriate IR regulator, one has to confirm that the IR divergences from the different diagrams cancel (“Kinoshita-Lee-Nauenberg cancellation”¹⁵⁾) or are completely factorized into parton distribution functions. Such a calculation has been performed for $\mathcal{O}(\alpha_s)$ corrections to tDY using various IR regularization schemes. For example, the one-loop calculation of the tDY cross section was done by Vogelsang and Weber¹⁶⁾ using the massive gluon scheme. The same calculation was also done using the dimensional reduction scheme.¹⁷⁾ The relations among the results obtained with different schemes are discussed in Ref. 18). The result in the dimensional regularization scheme was obtained¹⁹⁾ by using the scheme transformation relation (see also Ref. 20)). We also note that all these works^{16)–20)} investigating tDY treat the case in which the transverse momentum Q_T of the final dilepton $l^+ l^-$ with respect to the beam axis is unobserved (integrated).*)

When a calculation of such QCD corrections is performed, one encounters an interesting technical problem in the case of transverse polarization. As is well known,¹²⁾ the cross section depends on the azimuthal angle, ϕ , of the observed particle, and its dependence in tDY is of the form $\cos(2\phi)$. Therefore, we must keep the azimuthal angle dependence of the cross section in the case of transverse polarization, which makes it difficult to perform the phase space integrals in the higher-order calculations. This difficulty becomes much more severe if dimensional regularization is employed. One cannot use the techniques developed for the unpolarized and longitudinally polarized DY. A related problem in the dimensional regularization scheme has been discussed by Kamal,²¹⁾ but an explicit result was not given.

*) Ref. 16) treats also the case with Q_T observed, restricting Q_T to large values, which makes the IR regulator irrelevant for calculation of the $\mathcal{O}(\alpha_s)$ contributions to the cross section.

Recently, Mukherjee et al.²²⁾ proposed a new technique which allows one to overcome a similar problem in their calculation of prompt photon production.

In this paper, we employ the dimensional regularization scheme and discuss our approach to directly integrate out the phase space in D dimensions. The final result for the tDY cross section up to $\mathcal{O}(\alpha_s)$ accuracy in the $\overline{\text{MS}}$ scheme has been reported in a previous paper,²³⁾ and here we present the details of our calculation. We believe that our explicit calculation in the dimensional regularization is interesting and useful. Once the above-mentioned complication associated with the transverse polarization is worked out, the dimensional regularization provides us with a manifestly gauge-invariant and the most transparent framework to calculate the QCD corrections for both the Q_T -unobserved and Q_T -observed cases. For the latter case, in particular, we are able to derive the tDY cross section up to $\mathcal{O}(\alpha_s)$ without any restriction on Q_T , explicitly isolating the terms that diverge as $\ln(Q^2/Q_T^2)/Q_T^2$, $1/Q_T^2$, and $\delta(Q_T^2)$, as $Q_T \rightarrow 0$. The resulting “ Q_T -differential” cross section gives the leading order (LO) QCD prediction in the large Q_T region, in which $Q_T \sim Q$, where the lepton-pair production via the DY mechanism has to be accompanied by the radiation of at least one recoiling “hard” gluon, and for this reason the fixed-order truncation of perturbation theory is effective. On the other hand, the unlimited growth of the singular terms of the form $\ln(Q^2/Q_T^2)/Q_T^2$ in the cross section at small Q_T is associated with the recoil from “soft” gluon radiation, and this implies that we cannot truncate perturbation theory and have to calculate the higher-order QCD corrections beyond the one-loop level in this edge region of phase space.

The higher-order corrections that control the small Q_T behavior of the cross section can be taken into account through the soft gluon resummation (“transverse-momentum (Q_T) resummation”). As the next step, we derive the soft gluon resummation for our Q_T -differential tDY cross section, so that we can extend our LO QCD prediction at large Q_T to the entire range of Q_T . Below, we summarize the development of the soft gluon resummation.

A small value of Q_T ($\ll Q$) implies that there exists a new scale in the problem, and as a result, in the perturbative calculation, there appear terms containing large logarithms of Q^2/Q_T^2 : The coefficient of $\alpha_s^n(Q^2)$ includes a factor of $1/Q_T^2$ multiplying a series of logarithms of the form $\ln^m(Q^2/Q_T^2)$, with $m = 0, 1, \dots, 2n - 1$. The pattern of these logarithmic terms is characteristic of a theory with massless vector bosons, such as QCD and QED, and is produced by recoil from the radiation of gluons and photons. The first work dealing with these enhanced “recoil logarithms” in QCD was carried out by Dokshitzer, Dyakonov and Troyan.²⁴⁾ Their result corresponds to the leading logarithmic (LL) resummation in momentum space, i.e., the resummation of the terms $\alpha_s^n \ln^{2n-1}(Q^2/Q_T^2)$ to all orders in α_s . The level of this approximation is the same as that in Sudakov’s QED analysis,²⁵⁾ and the result was derived by imposing the “strong ordering” of the gluon’s transverse momenta

in the relevant Feynman diagrams; the strong ordering excessively constrains the phase space of the emitted soft gluons, and thus results in the transverse-momentum conservation being ignored. To take into account the transverse-momentum conservation, Parisi and Petronzio²⁶⁾ developed a formulation in the space of the impact parameter, b , which is the Fourier conjugate of the Q_T space. The relation between the Q_T -space and the b -space approaches is analyzed in Ref. 27). That work clarifies the impact of the transverse-momentum conservation on the subleading logarithmic terms. The general form of the soft gluon resummation to all orders of logarithms was proved and formulated in the b space approach by Collins and Soper.²⁸⁾ The universal two-loop anomalous dimension in the resummed cross sections, which is necessary for the next-to-leading logarithmic (NLL) analysis of *all* relevant processes, was first calculated in Ref. 29). This result was based on a plausible assumption, and Davies et al.³⁰⁾ confirmed the result using an explicit calculation of the unpolarized DY process up to order α_s^2 . They also carried out a phenomenological analysis, but only for small Q_T .

Advanced b -space formulations of the soft gluon resummation, which are suitable for phenomenological analyses of DY in all Q_T regions, were developed in the following works. Altarelli et al.³¹⁾ proposed a recipe to include the NLL resummation effects into the unpolarized DY cross section, but their approach is somewhat naive; an application of this approach to the longitudinally polarized DY process is considered in Ref. 33). Presently, the formulation which is valid to all orders of logarithms developed by Collins, Soper and Sterman (CSS)³²⁾ is regarded as standard. Recently, de Florian and Grazzini³⁴⁾ derived a universal expression for the CSS resummation formula up to next-to-next-to-leading logarithmic (NNLL) accuracy, which is applicable to DY production, electroweak boson production, Higgs boson production, etc. Further developments of formulations and their applications to phenomenology are underway (see Refs. 35)–37) and references therein).*) The extension of the formulations to DY with transversely polarized beams has been carried out by the present authors, and the main results are reported in Refs. 23) and 56).**)

The general formulation of soft gluon resummation used to obtain our results is only briefly described in Refs. 23) and 56). In this paper, we discuss the general form of the b -space resummation formula in detail from a modern viewpoint, emphasizing its theoretical basis as well as its physical content. We also demonstrate that the b space can be divided

*) Besides the Q_T resummation discussed in this paper, another kind of soft gluon resummation, the so-called “threshold resummation,” has been developed for the purpose of resumming the large higher-order corrections near the threshold of partonic scattering.^{41), 42), 53), 68)} Also, the joint resummation formalism has been devised to perform the Q_T and threshold resummations simultaneously.^{45)–47)}

**) See Ref. 38) for a treatment within the LL-level resummation. The extension to polarized semi-inclusive DIS has recently been discussed.³⁹⁾

into three distinct regions, associated with different distance scales; each of these three regions has to be treated differently, and the results for the three regions must eventually be combined in a consistent manner. This point is addressed in the original papers on the b -space approach,^{26),32)} but here we present a more systematic explanation of this point, which is important to ensure the maximal applicability of the resummation formalisms to a wide range of processes, including spin-dependent processes at moderate and high energies. Then, we apply the resummation formalism to our tDY Q_T -differential cross section, which allows us to include all orders resummation of the logarithmically-enhanced contributions for small Q_T due to multiple emissions of soft gluons in QCD. We perform the corresponding resummation up to NLL accuracy, and the result is combined with the fixed-order LO cross section that controls the large Q_T region, yielding a tDY cross section with uniform accuracy over the entire range of Q_T . We also explicitly derive the important properties of our cross section, which were briefly addressed in previous papers.^{23),56)} For example, our cross section satisfies the unitarity constraint exactly; the NLL resummation is controlled completely by the “saddle point” in the b space in the asymptotic regime, $Q \gg \Lambda_{\text{QCD}}$, $Q_T \approx 0$. As an application of our results, we calculate the dilepton Q_T spectrum and the cross section asymmetry as functions of Q_T . These quantities are to be observed in polarized pp collisions with large CM energy $\sqrt{S} \gtrsim 200$ GeV at RHIC. We also present the results to be observed in polarized pp collisions with moderate $\sqrt{S} = 10$ GeV at J-PARC. We demonstrate that, for both RHIC and J-PARC energies, the soft gluon resummation is crucial for making a reliable QCD prediction for the small Q_T region, where the bulk of dileptons is produced.

To our great sorrow, one of the present authors, Jiro Kodaira, died on September 16, 2006. The present work was performed by the three authors jointly, and many parts of this paper are based on our “collaboration notes,” originally written by Jiro. To complete this paper, we have had to reorganize and expand those notes without the direct assistance of Jiro, but we have been guided by his style in the approach to the problem, which we learned through our association with him for more than ten years.

The rest of the paper is organized as follows. Sections 2-4 are devoted to the calculation of the tDY cross section up to $\mathcal{O}(\alpha_s)$ in the dimensional regularization. Section 2 is mainly introductory, explaining the factorization theorem for the tDY cross section, and the relevant partonic mechanism to $\mathcal{O}(\alpha_s)$. The total and Q_T -differential cross sections are derived in §§3 and 4, respectively, by performing a collinear factorization in the $\overline{\text{MS}}$ scheme. Sections 5-9 contain discussion of the soft gluon resummation. In §5 we introduce the general formalism of soft gluon resummation, and in §6 we elaborate on its b -space structure. We perform the soft gluon resummation for tDY up to NLL accuracy in §7. In §8, we also derive an asymptotic formula for our NLL resummed cross section in the $Q_T \approx 0$ region. Section 9 contains

numerical results for tDY processes to be observed at RHIC and J-PARC. We demonstrate the roles of QCD soft gluon effects in the cross sections and asymmetries as functions of Q_T , and discuss a new approach to extracting the transversity from experimental data. The final section, §10, is reserved for conclusions. This paper contains three appendices. In Appendix A, we collect the operator definitions and basic properties of transversity, in Appendix B we collect the formulae for the tDY cross sections integrated over the rapidity of the dilepton, and Appendix C supplements the discussion given in §7.

§2. Drell-Yan mechanism to order α_s

2.1. Factorization formula for the transversely polarized Drell-Yan process

The process we consider is the tDY process,

$$h_1(P_1, S_1) + h_2(P_2, S_2) \rightarrow l(k_1) + \bar{l}(k_2) + X, \quad (2.1)$$

where h_1 and h_2 denote spin-1/2 hadrons with momenta P_1 and P_2 and transverse spins S_1 and S_2 , and $Q = k_1 + k_2$ is the 4-momentum of the DY pair. The tDY (2.1) may be induced by partonic subprocesses, such as quark-antiquark annihilation,

$$q(p_1, s_1) + \bar{q}(p_2, s_2) \rightarrow l(k_1) + \bar{l}(k_2) + X, \quad (2.2)$$

where q and \bar{q} have momenta p_1 and p_2 and transverse spins s_1 and s_2 . This can be formulated precisely using QCD factorization,¹⁾ which is valid when Q^2 is large, i.e., $Q^2 \gg \Lambda_{\text{QCD}}^2$, with $\tau \equiv Q^2/S$ ($S \equiv (P_1 + P_2)^2$) fixed. In this case, (2.2) and $\bar{q}(p_1, s_1) + q(p_2, s_2) \rightarrow l(k_1) + \bar{l}(k_2) + X$ provide the dominant mechanism for tDY in the region with $p_i = x_i P_i$ ($i = 1, 2$), where x_1 and x_2 denote the longitudinal-momentum fractions of partons inside the parent hadrons h_1 and h_2 , respectively, while the contributions from the other regions or other partonic processes give subleading corrections. The values that x_1 or x_2 actually takes are determined by the parton distributions, due to the long-distance, nonperturbative dynamics inside each hadron $h_{1,2}$, whose typical distance scale $\sim 1/\Lambda_{\text{QCD}}$ is well-separated from the short-distance scale of order $1/Q$, within which partonic annihilation processes like (2.2) occur. Therefore, the short- and long-distance contributions can be treated separately, and the spin-dependent cross section, $\Delta_T d\sigma \equiv (d\sigma(S_1, S_2) - d\sigma(S_1, -S_2))/2$, for tDY (2.1) is given as a convolution of the short- and long-distance contributions, over each of the partonic momentum fractions x_1 and x_2 for the two hadrons $h_{1,2}$,^{12),13)}

$$\Delta_T d\sigma = \int dx_1 dx_2 \delta H(x_1, x_2; \mu_F^2) [\Delta_T d\hat{\sigma}(p_1, p_2; Q^2; \mu_F^2)]_{p_i = x_i P_i}, \quad (2.3)$$

up to the higher-twist corrections suppressed by the powers of $1/Q$. Here, μ_F is the factorization scale, $\Delta_T d\hat{\sigma}(p_1, p_2; Q^2; \mu_F^2)$ is the transversely polarized partonic cross section for (2.2) representing the short-distance contribution, and

$$\delta H(x_1, x_2; \mu_F^2) = \sum_q e_q^2 [\delta q_{h_1}(x_1, \mu_F^2) \delta \bar{q}_{h_2}(x_2, \mu_F^2) + \delta \bar{q}_{h_1}(x_1, \mu_F^2) \delta q_{h_2}(x_2, \mu_F^2)] \quad (2.4)$$

is the long-distance contribution as a product of the transversity distributions^{12), 13)} $\delta q_{h_1}(x_1, \mu_F^2)$ and $\delta q_{h_2}(x_2, \mu_F^2)$ for the two colliding hadrons, h_1 and h_2 , summed over the massless quark flavors q with their charge squared e_q^2 .

Intuitively, the leading-twist factorization formula (2.3) can be interpreted using the language of the parton model, similarly to the factorization formula for the twist-2 structure functions in DIS. In particular, the transversity distribution $\delta q_h(x, \mu_F^2)$ is one of the three independent twist-2 quark distribution functions for a spin-1/2 hadron h , which are associated with the probability distribution of finding a quark with a longitudinal momentum fraction x inside the parent hadron h . In fact, $\delta q_h(x, \mu_F^2)$ is defined by^{12)–14)}

$$\delta q_h(x, \mu_F^2) \equiv q_h^\uparrow(x, \mu_F^2) - q_h^\downarrow(x, \mu_F^2) ,$$

where q_h^\uparrow (q_h^\downarrow) denotes the probability distribution of transversely polarized quarks with spin parallel (antiparallel) to the spin of their transversely-polarized parent hadron.¹⁴⁾ The spin-dependent partonic cross section participating in (2.3) is defined as

$$\begin{aligned} \Delta_T d\hat{\sigma}(p_1, p_2; Q^2; \mu_F^2) \equiv & \frac{1}{2} [d\hat{\sigma}(p_1, p_2; s_1, s_2; Q^2; \mu_F^2) \\ & - d\hat{\sigma}(p_1, p_2; s_1, -s_2; Q^2; \mu_F^2)] , \end{aligned} \quad (2.5)$$

in terms of the partonic hard cross section $d\hat{\sigma}(p_1, p_2; s_1, s_2; Q^2; \mu_F^2)$ which describes the annihilation process (2.2) of the quark and antiquark with the transverse spins s_1 and s_2 , respectively. Note that at the leading twist level, the gluon distributions do not contribute to (2.3) for the transversely polarized process, due to its chiral-odd nature.^{12)–14)} Therefore, only the $q\bar{q}$ annihilation subprocesses give the relevant twist-2 mechanism for the spin-dependent cross section in tDY.

The factorization formula (2.3) provides not only a realization of the parton model based on QCD but also a systematic framework for calculating the QCD corrections beyond the parton model. As mentioned above, the meaning of the factorization formula (2.3) is the separation of the short- and long-distance contributions contained in the process (2.1) at the factorization scale μ_F . In formal field theoretical language, (2.1) can be represented by Feynman diagrams, assuming that the hadrons $h_{1,2}$ can be described in terms of virtual

partonic states, as if they were a beam of quarks and gluons. Then, in a corresponding generic diagram, the long-distance contributions may give rise to infrared (IR) divergence, but, considering the set of diagrams contributing at the same order of accuracy, those IR-divergent contributions can be completely factorized into universal hadron matrix elements [transversity distributions, see (A.1)], or cancel out, and thus the partonic cross section (2.5) possesses purely short-distance contributions containing all the dependence on the relevant hard scale Q .¹⁾ Therefore, QCD perturbation theory can provide a prediction for (2.5) in a systematic way, while the transversity distributions as the universal hadron matrix elements can only be determined by taking into account nonperturbative effects.

A traditional approach for the separation of short- and long-distance contributions is the operator product expansion (OPE), and in fact, the OPE has been successfully applied to, e.g., deep inelastic lepton-hadron scattering. But, in the case of DY processes, as well as various hard processes in which two or more hadrons participate, the conventional OPE in terms of local operators is useless for this purpose, and the factorization theorems associated with formulae like (2.3) provide a generalization of the OPE suitable for those processes.¹⁾ The partonic cross section (2.5) in the factorization formula (2.3) plays the role of the Wilson coefficient functions in the conventional OPE, and (2.5) can be determined by a “matching” procedure similar to that used to obtain the Wilson coefficient functions in the OPE: To obtain the prediction for (2.5), we apply the factorization formula (2.3) to the parton-level process (2.2), employing an appropriate IR regularization scheme. In this case, (2.4) can be calculated explicitly as the “quark distribution inside a quark,” $\delta q_{h_1=q}(x_1, \mu_F^2) = \delta(x_1 - 1) + \dots$ and $\delta \bar{q}_{h_2=\bar{q}}(x_2, \mu_F^2) = \delta(x_2 - 1) + \dots$, where the ellipses represent the terms of $\mathcal{O}(\alpha_s)$ or higher, depending on the specific IR regulator [see (A.3) in Appendix A]. The IR-regulator-dependent contributions on the RHS of (2.3) actually coincide with the IR-regulator-dependent contributions appearing on the LHS for the parton-level cross section, and thus a comparison of both sides of (2.3), order-by-order in α_s , yields $\Delta_T d\hat{\sigma}(p_1, p_2; Q^2; \mu_F^2)$ of (2.5) as an “IR safe” power series in α_s . Through this matching procedure, we confirm that the partonic cross section, $\Delta_T d\hat{\sigma}(p_1, p_2; Q^2; \mu_F^2)$, indeed represents a purely short-distance component in the cross section (2.3). In our calculation presented below, we employ the dimensional regularization scheme to regularize IR and UV divergences.

2.2. Momentum and spin for parton-level processes in D dimensions

We now perform the above-mentioned matching at the one-loop level and derive $\Delta_T d\hat{\sigma}(p_1, p_2; Q^2; \mu_F^2)$ in (2.5) to $\mathcal{O}(\alpha_s)$. For this purpose, we calculate the LHS of (2.3) for the parton-level process (2.2), which we write as $\Delta_T d\sigma^{q\bar{q}}$. At the one-loop level, the diagrams to be calculated are given in Fig. 1. We calculate all these contributions in $D = 4 - 2\epsilon$ dimensions to regulate

the IR and UV divergences. For these parton-level processes, we define the invariants as

$$s = (p_1 + p_2)^2, \quad Q^2 = (k_1 + k_2)^2 \equiv z s, \quad t = (p_1 - k_1)^2, \quad u = (p_2 - k_1)^2, \quad (2.6)$$

in terms of the momenta assigned in (2.2). In the case of one-gluon emission corresponding to the lower diagrams in Fig. 1, i.e., (2.2) with $X \equiv g(k)$, we also define

$$\hat{t} = (p_1 - k)^2, \quad \hat{u} = (p_2 - k)^2. \quad (2.7)$$

The momentum of a massless particle, $p = \{p_1, p_2, k_1, k_2, k\}$, in D dimensions is generically expressed as

$$p^\mu = (p^0, \mathbf{p}) = (|\mathbf{p}|, \mathbf{p}), \quad \mathbf{p} \equiv |\mathbf{p}|(\mathbf{n}_p^{(D-2)} \sin \theta_p, \cos \theta_p), \quad (2.8)$$

where $\mathbf{n}_p^{(D-2)}$ denotes a $(D-2)$ -dimensional unit vector which is parametrized by $D-3$ angular coordinates, $\phi_p^{(D-3)}, \phi_p^{(D-4)}, \dots$, as

$$\begin{aligned} \mathbf{n}_p^{(D-2)} &= (\cos \phi_p^{(D-3)}, \mathbf{n}_p^{(D-3)} \sin \phi_p^{(D-3)}) \\ &= (\cos \phi_p^{(D-3)}, \sin \phi_p^{(D-3)} \cos \phi_p^{(D-4)}, \mathbf{n}_p^{(D-4)} \sin \phi_p^{(D-3)} \sin \phi_p^{(D-4)}) \\ &= \dots, \end{aligned} \quad (2.9)$$

such that in 4 dimensions, defining $\phi_p^{(1)} \equiv \phi_p$, we have

$$\mathbf{n}_p^{(2)} = (\cos \phi_p, \sin \phi_p). \quad (2.10)$$

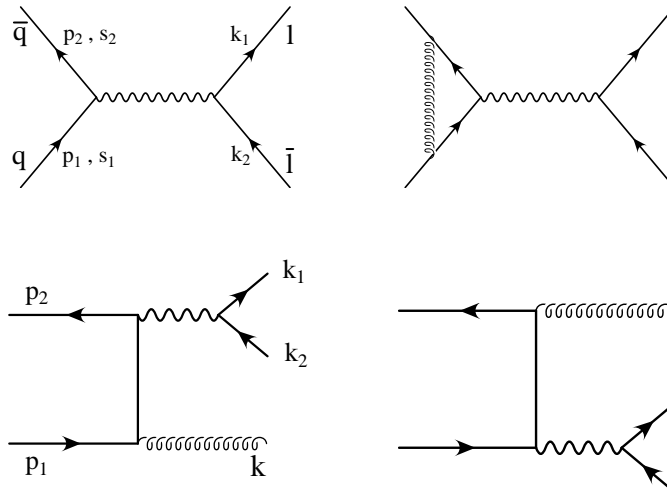


Fig. 1. Parton-level processes contributing to the spin-dependent cross section in tDY to $\mathcal{O}(\alpha_s)$.

When we calculate the LHS of (2.3) for the parton-level process (2.2), we always choose a frame in which the initial quark and antiquark in (2.2) possess momenta \mathbf{p}_1 and \mathbf{p}_2 that are collinear along the z -axis, with $\theta_{p_1} = 0$ and $\theta_{p_2} = \pi$, and possess transverse spins, satisfying $p_1 \cdot s_1 = 0$, $p_2 \cdot s_2 = 0$, and $s_{1,2}^2 = -1$, that are expressed by

$$s_{1,2}^\mu = (0, \mathbf{s}_{1,2}^{(D-2)}, 0), \quad (2.11)$$

with the $(D-2)$ -dimensional unit vectors $\mathbf{s}_{1,2}^{(D-2)}$, which reduce in 4 dimensions to

$$\mathbf{s}_1^{(2)} = (\cos \phi_1, \sin \phi_1), \quad \mathbf{s}_2^{(2)} = (\cos \phi_2, \sin \phi_2). \quad (2.12)$$

2.3. Parton-level amplitude squared

The contribution of each diagram in Fig. 1 to the spin-dependent cross section $\Delta_T d\sigma^{q\bar{q}}$, the LHS of (2.3), is generically given by

$$\frac{1}{2N_c^2 s} \Delta_T |M|^2 d\Phi, \quad (2.13)$$

where $d\Phi$ denotes the differential element of the n -body phase space for the corresponding $2 \rightarrow n$ process ($n = 2, 3$), $1/N_c$ is the color-averaging factor for the incoming quark (antiquark), and the spin-dependent amplitude squared, $\Delta_T |M|^2$, is defined in terms of the relevant $2 \rightarrow n$ Feynman amplitude $M(s_1, s_2)$, associated with the transverse spins s_1 and s_2 for the colliding quark and antiquark in (2.2), as

$$\Delta_T |M|^2 \equiv \frac{1}{2} [|M(s_1, s_2)|^2 - |M(s_1, -s_2)|^2], \quad (2.14)$$

where the sum over the spins of the final-state leptons is implicit. We calculate (2.13) and (2.14) in the dimensional regularization. We rewrite the QCD coupling constant as $g \rightarrow g\mu^\epsilon$ with the mass scale μ . This enables us to keep the redefined coupling constant dimensionless in $D = 4 - 2\epsilon$ dimensions.^{*)}

In the spin-dependent amplitude squared of (2.14), we encounter traces of the gamma matrices involving γ_5 . We employ the naive anticommuting- γ_5 scheme, $\{\gamma_5, \gamma^\mu\} = 0$ ($\mu = 0, 1, \dots, D-1$), which is the usual prescription in the transverse-spin channel: For the transverse-spin case, it appears that these traces involve only even numbers of γ_5 . Therefore the matrices γ_5 eventually disappear due to $\gamma_5^2 = 1$, and we do not anticipate any inconsistencies related to γ_5 . This suggests that the naive anticommuting- γ_5 prescription will work (see Ref. 19)).

^{*)} In principle, one can rewrite the QED coupling constant similarly. This results in the multiplication of the Born cross section and the higher-order cross section by a common factor of $\mu^{4\epsilon}$, and it allows us to keep the dimensions of the cross section unchanged. But this is irrelevant for the present purpose of matching to obtain the partonic cross section (2.5).

The upper two diagrams in Fig. 1 represent the $2 \rightarrow 2$ processes up to $\mathcal{O}(\alpha_s)$. The corresponding tree-plus-virtual contributions to (2.14) can be calculated straightforwardly, and they read

$$\begin{aligned} \Delta_T |M|_{\text{T+V}}^2 &= 4 N_c \left(\frac{4\pi\alpha e_q}{s} \right)^2 \left[2 s (k_1 \cdot s_1)(k_1 \cdot s_2) + t u (s_1 \cdot s_2) + \epsilon \frac{s^2}{2} (s_1 \cdot s_2) \right] \\ &\times \left[1 + \frac{\alpha_s}{\pi} C_F \left(\frac{4\pi\mu^2}{s} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left(-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - 4 + \frac{\pi^2}{2} + \mathcal{O}(\epsilon) \right) \right], \end{aligned} \quad (2.15)$$

where $C_F = (N_c^2 - 1)/(2N_c)$ [also see (2.6)]. Equation (2.15) includes the contribution due to the wave-function renormalization factors for the incoming quark and antiquark legs, which cancels the quark-photon vertex renormalization constant, as guaranteed by the Ward identity. Therefore, (2.15) is UV finite, and it is also gauge invariant in a gauge-invariant regularization scheme. Note that the ratio of the $\mathcal{O}(\alpha_s)$ term to the $\mathcal{O}(1)$ term in (2.15) is the same as the corresponding ratio in the unpolarized case.⁴⁰⁾

The lower two diagrams in Fig. 1 represent the $2 \rightarrow 3$ processes up to $\mathcal{O}(\alpha_s)$. The relevant polarization sum for the final-state gluon, $\mathcal{P}^{\alpha\beta}(k) \equiv \sum_\lambda e_{(\lambda)}^\alpha(k) e_{(\lambda)}^{*\beta}(k)$, where $e_{(\lambda)}^\alpha(k)$ is the polarization vector for the real gluon with momentum k and the physical polarization λ , can be replaced as $\mathcal{P}^{\alpha\beta}(k) \rightarrow -g^{\alpha\beta}$, using the Ward identity, in the present case of *one* real external massless gluon.⁴³⁾ We write the corresponding result for the one-gluon-radiation contributions as

$$\Delta_T |M|_{\text{R}}^2 \equiv N_c C_F \left(\frac{4\pi\alpha e_q g \mu^\epsilon}{Q^2} \right)^2 \Delta_T \mathcal{R}, \quad (2.16)$$

where $\Delta_T \mathcal{R}$ is formally given by a second-order polynomial in ϵ , after working out the traces of the Dirac matrices. Actually, the terms proportional to ϵ^2 in $\Delta_T \mathcal{R}$ do not contribute to the cross section in the limit of $\epsilon \rightarrow 0$, and therefore we drop them. Similarly, we drop some of the terms proportional to ϵ , which clearly do not contribute to the cross section, and we thereby obtain

$$\begin{aligned} \Delta_T \mathcal{R} &= \frac{16\epsilon}{\hat{t}^2} [s(zt + u + Q^2)(k_1 \cdot s_2)(k \cdot s_1) - tu(k \cdot s_1)(k \cdot s_2)] \\ &+ \frac{8\epsilon}{\hat{t}} [2(s - Q^2)(k_1 \cdot s_1)(k_1 \cdot s_2) + u(t + u + Q^2)(s_1 \cdot s_2)] \\ &+ \frac{16\epsilon}{\hat{u}^2} [s(zu + t + Q^2)(k_1 \cdot s_1)(k \cdot s_2) - tu(k \cdot s_1)(k \cdot s_2)] \\ &+ \frac{8\epsilon}{\hat{u}} [2(s - Q^2)(k_1 \cdot s_1)(k_1 \cdot s_2) + t(u + t + Q^2)(s_1 \cdot s_2)] \\ &+ \frac{16Q^2}{\hat{t}\hat{u}} \left[2s(k_1 \cdot s_1)(k_1 \cdot s_2) + tu(s_1 \cdot s_2) + \epsilon \frac{Q^2 s}{2} (s_1 \cdot s_2) \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{8Q^2}{\hat{t}\hat{u}} [2s \{ (k_1 \cdot s_2)(k \cdot s_1) + (k_1 \cdot s_1)(k \cdot s_2) + (k \cdot s_1)(k \cdot s_2) \} \\
& \quad + \{ Q^2 (s + t + u) - t\hat{t} - u\hat{u} + \hat{t}\hat{u} \} (s_1 \cdot s_2)] \\
& + \frac{8\epsilon}{\hat{t}\hat{u}} s (t + u + Q^2) (s + t + u) (s_1 \cdot s_2) \\
& + \frac{4\epsilon}{\hat{t}} [t (s + t + u) + (s + t) (t + u + Q^2)] (s_1 \cdot s_2) \\
& + \frac{4\epsilon}{\hat{u}} [u (s + t + u) + (s + u) (u + t + Q^2)] (s_1 \cdot s_2), \tag{2.17}
\end{aligned}$$

using the variables in (2.6) and (2.7). Note that the terms in (2.17) may receive additional factors of $1/\epsilon$ as IR-divergent poles through the integration over the phase space in (2.13).

§3. NLO matching for the Q_T -integrated (total) cross section

First, we consider the case in which the transverse momentum, Q_T , of the final dilepton is unobserved (integrated) in the parton-level process (2.2). We calculate the corresponding cross section (2.3) using (2.13)-(2.17) and determine the Q_T -integrated partonic cross section, $\Delta_T d\hat{\sigma}(p_1, p_2; Q^2; \mu_F^2)$, to $\mathcal{O}(\alpha_s)$.

3.1. Tree-plus-virtual contributions

The phase space for an outgoing massless particle with momentum p is given by [see (2.8) and (2.9)]

$$\frac{d^{D-1}p}{(2\pi)^{D-1}2p^0} = \frac{1}{2(2\pi)^{D-1}} |\mathbf{p}|^{D-3} d|\mathbf{p}| d\Omega_p^{(D-1)},$$

where the differential element for the $(D-2)$ -dimensional angular integration is expressed as

$$d\Omega_p^{(D-1)} = (1 - \cos^2 \theta_p)^{-\epsilon} d\cos \theta_p d\Omega_p^{(D-2)}. \tag{3.1}$$

The two-particle phase space for the $2 \rightarrow 2$ processes of Fig. 1 is given by

$$d\Phi_2 = \frac{d^{D-1}k_1}{(2\pi)^{D-1}2k_1^0} \frac{d^{D-1}k_2}{(2\pi)^{D-1}2k_2^0} (2\pi)^D \delta^{(D)}(p_1 + p_2 - k_1 - k_2), \tag{3.2}$$

and we obtain for the tree-plus-virtual contributions to (2.13) the expression

$$\frac{\Delta_T d\sigma_{T+V}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-1)}} = \frac{1}{64\pi^2 N_c^2 s^2} \left(\frac{\sqrt{s}}{4\pi} \right)^{-2\epsilon} \Delta_T |M|_{T+V}^2 \delta(1-z),$$

using the invariants of (2.6). Inserting the expression for $\Delta_T |M|_{T+V}^2$, (2.15), and using the explicit forms (2.8)-(2.11) for the lepton's momentum k_1 and the quark's transverse spin $s_{1,2}$,

we obtain

$$\begin{aligned}
\frac{\Delta_T d\sigma_{\text{T+V}}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-1)}} &= \frac{1}{16\pi^2 N_c} \left(\frac{4\pi\alpha e_q}{Q^2} \right)^2 \left(\frac{Q^2}{16\pi^2} \right)^{-\epsilon} \delta(1-z) \\
&\times \left[\frac{1}{4} \alpha_\Phi \sin^2 \theta_{k_1} + \frac{\epsilon}{2} (s_1 \cdot s_2) \right] \\
&\times \left[1 + \frac{\alpha_s}{\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left(-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - 4 + \frac{\pi^2}{2} + \mathcal{O}(\epsilon) \right) \right],
\end{aligned} \tag{3.3}$$

where

$$\alpha_\Phi = 2 (\mathbf{n}_{k_1}^{(D-2)} \cdot \mathbf{s}_1^{(D-2)}) (\mathbf{n}_{k_1}^{(D-2)} \cdot \mathbf{s}_2^{(D-2)}) + s_1 \cdot s_2, \tag{3.4}$$

with

$$s_1 \cdot s_2 = -\mathbf{s}_1^{(D-2)} \cdot \mathbf{s}_2^{(D-2)}. \tag{3.5}$$

From (3.1), the corresponding “azimuthal” angular distribution of the lepton is obtained by integrating (3.3) over the “scattering” angle θ_{k_1} , with the measure $(1 - \cos^2 \theta_{k_1})^{-\epsilon} d \cos \theta_{k_1}$, as

$$\begin{aligned}
\frac{\Delta_T d\sigma_{\text{T+V}}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}} &= \frac{1}{16\pi^2 N_c} \left(\frac{4\pi\alpha e_q}{Q^2} \right)^2 \left(\frac{Q^2}{16\pi^2} \right)^{-\epsilon} \delta(1-z) \\
&\times \left[2^{1-2\epsilon} B(2-\epsilon, 2-\epsilon) \alpha_\Phi + \epsilon 2^{-2\epsilon} B(1-\epsilon, 1-\epsilon) (s_1 \cdot s_2) \right] \\
&\times \left[1 + \frac{\alpha_s}{\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left(-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - 4 + \frac{\pi^2}{2} + \mathcal{O}(\epsilon) \right) \right],
\end{aligned} \tag{3.6}$$

where $B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$.

3.2. Real gluon emission

For the $2 \rightarrow 3$ processes depicted in Fig. 1, the real-gluon emission contributions, the corresponding amplitude squared seems rather complicated [see (2.17)]. Fortunately, miraculous cancellations occur among the first four lines and among the last three lines of (2.17):²¹⁾ Because the corresponding terms are proportional to ϵ , these terms survive only in the configuration obtained in the $\hat{t}, \hat{u} \rightarrow 0$ limit, in which the divergences associated with the collinear ($k^\mu \propto p_1^\mu$, $k^\mu \propto p_2^\mu$) or soft ($k^\mu \approx 0$) gluon are produced [see (2.7)]. An explicit calculation shows that the collinear configuration is indeed relevant in the present case, and in this configuration an additional $1/\epsilon$ factor multiplying these terms appears. However, for such collinear configurations, it turns out that the first (third) line cancels the second

(fourth) line in (2·17), and also, the last three lines cancel. Therefore, (2·17) can be reduced to the very simple form

$$\Delta_T \mathcal{R} = \Delta_T \mathcal{R}_S + \Delta_T \mathcal{R}_{NS}, \quad (3·7)$$

with

$$\begin{aligned} \Delta_T \mathcal{R}_S &= \frac{16Q^2}{\hat{t}\hat{u}} \left[2s(k_1 \cdot s_1)(k_1 \cdot s_2) + t u (s_1 \cdot s_2) + \epsilon \frac{Q^2 s}{2} (s_1 \cdot s_2) \right], \\ \Delta_T \mathcal{R}_{NS} &= \frac{8Q^2}{\hat{t}\hat{u}} \left[2s \{ (k_1 \cdot s_2)(k \cdot s_1) + (k_1 \cdot s_1)(k \cdot s_2) + (k \cdot s_1)(k \cdot s_2) \} \right. \\ &\quad \left. + \{ Q^2 (s + t + u) - t\hat{t} - u\hat{u} + \hat{t}\hat{u} \} (s_1 \cdot s_2) \right]. \end{aligned} \quad (3·8)$$

We have divided the RHS of (3·7) into two terms; as will be discussed below, only $\Delta_T \mathcal{R}_S$, given in (3·8), receives the $1/\epsilon$ singularity through the integration over the phase space in (2·13), while $\Delta_T \mathcal{R}_{NS}$ yields the result finite in the limit $\epsilon \rightarrow 0$.*)

The three-particle phase space for the $2 \rightarrow 3$ processes depicted in Fig. 1 is given by

$$d\Phi_3 = \frac{d^{D-1}k_1}{(2\pi)^{D-1}2k_1^0} \frac{d^{D-1}k_2}{(2\pi)^{D-1}2k_2^0} \frac{d^{D-1}k}{(2\pi)^{D-1}2k^0} (2\pi)^D \delta^{(D)}(p_1 + p_2 - k - k_1 - k_2). \quad (3·9)$$

To use this in (2·13), it is convenient to employ the $q\bar{q}$ CM frame and first determine the phase space factor corresponding to the differential elements, $dQ^2 d\Omega_k^{(D-1)} d\Omega_{k_1}^{(D-1)}$, integrating over the other degrees of freedom [see (2·6)]. This results in the replacement

$$d\Phi_3 \rightarrow \frac{1}{8\sqrt{s}(2\pi)^{5-4\epsilon}} \frac{Q^{2(1-2\epsilon)}}{2^{2-2\epsilon}} \frac{|\mathbf{k}|^{1-2\epsilon}}{(Q^0 + \mathbf{k} \cdot \mathbf{k}_1/|\mathbf{k}_1|)^{2-2\epsilon}} dQ^2 d\Omega_k^{(D-1)} d\Omega_{k_1}^{(D-1)}, \quad (3·10)$$

where

$$Q^0 = \frac{\sqrt{s}}{2} (1 + z), \quad |\mathbf{k}| = \frac{\sqrt{s}}{2} (1 - z),$$

and the magnitude of the lepton momentum, $k_1^\mu \equiv (|\mathbf{k}_1|, \mathbf{k}_1)$, is fixed by the relevant kinematical constraints as

$$|\mathbf{k}_1| = \frac{Q^2}{2} \frac{1}{Q^0 + \mathbf{k} \cdot \mathbf{k}_1/|\mathbf{k}_1|} \equiv \frac{Q^2}{\sqrt{s}} \frac{1}{\mathcal{D}_I}, \quad (3·11)$$

with [see (2·8)]

$$\mathcal{D}_I = 1 + z + (1 - z) \left(\cos \theta_k \cos \theta_{k_1} + \sin \theta_k \sin \theta_{k_1} \mathbf{n}_k^{(D-2)} \cdot \mathbf{n}_{k_1}^{(D-2)} \right), \quad (3·12)$$

which is positive definite. The invariants in (3·8) are now given by [see (2·6), (2·7)]

$$\begin{aligned} \hat{t} &= -\frac{s}{2} (1 - z) (1 - \cos \theta_k), & \hat{u} &= -\frac{s}{2} (1 - z) (1 + \cos \theta_k), \\ t &= -\sqrt{s} |\mathbf{k}_1| (1 - \cos \theta_{k_1}), & u &= -\sqrt{s} |\mathbf{k}_1| (1 + \cos \theta_{k_1}). \end{aligned} \quad (3·13)$$

*) Note that we have $k \cdot s_{1,2} \rightarrow 0$ and $Q^2(s + t + u) - t\hat{t} - u\hat{u} + \hat{t}\hat{u} \rightarrow 0$ in the collinear limit, that is, $k^\mu \parallel p_1^\mu$ or $k^\mu \parallel p_2^\mu$.

Substituting the above results, the cross section (2.13) for one-gluon emission in the $q\bar{q}$ CM frame reads

$$\frac{\Delta_T d\sigma_{\text{R}}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-1)}} = G(\epsilon) \int \Delta_T \mathcal{R} \frac{(1-z)^{1-2\epsilon}}{\mathcal{D}_I^{2-2\epsilon}} d\Omega_k^{(D-1)}, \quad (3.14)$$

where we have

$$G(\epsilon) = \frac{Q^2}{64\pi^2 N_c s^2} \left(\frac{4\pi\alpha e_q}{Q^2} \right)^2 \left(\frac{Q^2}{16\pi^2} \right)^{-\epsilon} \frac{\alpha_s}{\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{\pi^\epsilon}{4\pi}.$$

With (3.11), (3.13), (3.4), and (3.5), the singular part of $\Delta_T \mathcal{R}$, i.e., $\Delta_T \mathcal{R}_S$ given in (3.8), reads

$$\begin{aligned} \Delta_T \mathcal{R}_S &= \frac{16 Q^2}{\hat{t}\hat{u}} \left[\frac{Q^4}{\mathcal{D}_I^2} \alpha_\Phi \sin^2 \theta_{k_1} + \epsilon \frac{Q^2 s}{2} s_1 \cdot s_2 \right] \\ &= \frac{64 Q^2}{s^2} \frac{1}{(1-z)^2 \sin^2 \theta_k} \left[\frac{Q^4}{\mathcal{D}_I^2} \alpha_\Phi \sin^2 \theta_{k_1} + \epsilon \frac{Q^2 s}{2} s_1 \cdot s_2 \right]. \end{aligned}$$

Using (3.1) for the angular integration of the gluon in (3.14), and changing the integration variable $\cos \theta_k$ to $z_1 \equiv (1 + \cos \theta_k) / 2$, we obtain the contribution to (3.14) from the singular part of one-gluon emission as

$$\begin{aligned} \frac{\Delta_T d\sigma_{\text{R}(S)}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-1)}} &= G(\epsilon) \frac{32 Q^2}{s^2} \frac{2^{-2\epsilon}}{(1-z)^{1+2\epsilon}} \int_0^1 dz_1 \int d\Omega_k^{(D-2)} \left[\frac{z_1^{-\epsilon}}{(1-z_1)^{1+\epsilon}} + \frac{(1-z_1)^{-\epsilon}}{z_1^{1+\epsilon}} \right] \\ &\quad \times \left[\frac{Q^4}{\mathcal{D}_I^{4-2\epsilon}} \alpha_\Phi \sin^2 \theta_{k_1} + \epsilon \frac{Q^2 s}{2 \mathcal{D}_I^{2-2\epsilon}} s_1 \cdot s_2 \right]. \end{aligned} \quad (3.15)$$

Because \mathcal{D}_I depends on the angles of both the gluon and lepton as expressed in (3.12), it is difficult to work out the integration of (3.15) in $D = 4 - 2\epsilon$ dimensions. However, fortunately we can carry out the integration by noting the well-known formula

$$\frac{1}{(1-z)^{1+\epsilon}} = -\frac{1}{\epsilon} \delta(1-z) + \frac{1}{(1-z)_+} - \epsilon \left(\frac{\ln(1-z)}{1-z} \right)_+ + \mathcal{O}(\epsilon^2), \quad (3.16)$$

and the similar formula obtained through the replacement $z \rightarrow 1-z$ in (3.16); these formulae allow us to make manifest the singularities appearing in the limit $\epsilon \rightarrow 0$. Here the “+” distributions are defined, as usual, as

$$\begin{aligned} \int_0^1 dz \frac{f(z)}{(1-z)_+} &\equiv \int_0^1 dz \frac{f(z) - f(1)}{1-z}, \\ \int_0^1 dz f(z) \left(\frac{\ln(1-z)}{1-z} \right)_+ &\equiv \int_0^1 dz [f(z) - f(1)] \frac{\ln(1-z)}{1-z}, \end{aligned} \quad (3.17)$$

for a smooth test function $f(z)$. Note that the $1/\epsilon$ poles proportional to $\delta(1-z_1)$ and $\delta(z_1)$, when employing the formula (3.16), are associated with the radiation of the collinear gluon

from an initial particle as $z_1 = (1 + \cos \theta_k) / 2 = 1$ or 0 , while the $1/\epsilon$ pole proportional to $\delta(1 - z)$ is associated with the radiation of the soft gluon at the threshold for the reaction, $z = Q^2/s = 1$. After inserting (3.16) into (3.15), we need to carry out the integration in arbitrary $D = 4 - 2\epsilon$ dimensions only for the terms proportional to $1/\epsilon^2$ and $1/\epsilon$, associated with the following limiting cases for \mathcal{D}_I :

$$\mathcal{D}_I(z \rightarrow 1) = 2 , \quad (3.18)$$

$$\mathcal{D}_I(z_1 \rightarrow 1) = 1 + z + (1 - z) \cos \theta_{k_1} , \quad (3.19)$$

$$\mathcal{D}_I(z_1 \rightarrow 0) = 1 + z - (1 - z) \cos \theta_{k_1} . \quad (3.20)$$

Then, the corresponding angular integrals in (3.15) can be performed exactly. The integration of the remaining terms in (3.15), which are finite as $\epsilon \rightarrow 0$, can be performed in $D = 4$ dimensions, using [see (2.10)]

$$\mathcal{D}_I(D \rightarrow 4) = 1 + z + (1 - z) (\cos \theta_k \cos \theta_{k_1} + \sin \theta_k \sin \theta_{k_1} \cos(\phi_k - \phi_{k_1})) . \quad (3.21)$$

The corresponding angular integrals can be done straightforwardly, by using the formulae in Appendix A of Ref. 16). Therefore, the phase space integrals in (3.15) are tractable exactly up to corrections that vanish as $\epsilon \rightarrow 0$.

We write the contribution to (3.14) from the nonsingular part of $\Delta_T \mathcal{R}$, i.e., $\Delta_T \mathcal{R}_{NS}$ of (3.8), as $\Delta_T d\sigma_{R(NS)}^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-1)}$. Similarly to the finite terms in (3.15) discussed just above, the relevant angular integrals in $\Delta_T d\sigma_{R(NS)}^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-1)}$ can be calculated in $D = 4$ dimensions, where $\Delta_T \mathcal{R}_{NS}$ reduces to

$$\begin{aligned} \Delta_T \mathcal{R}_{NS} &= \frac{32 Q^2}{(1 - z)^2 \sin^2 \theta_k} \left[\frac{(1 - z)^2}{2} \sin^2 \theta_k \cos(\phi_k - \phi_1) \cos(\phi_k - \phi_2) \right. \\ &\quad + z(1 - z) \frac{\sin \theta_k \sin \theta_{k_1}}{\mathcal{D}_I} \{ \cos(\phi_k - \phi_1) \cos(\phi_{k_1} - \phi_2) + (1 \leftrightarrow 2) \} \\ &\quad \left. - \left\{ z + \frac{(1 - z)^2}{4} \sin^2 \theta_k - z(1 + z) \frac{1}{\mathcal{D}_I} - z(1 - z) \frac{\cos \theta_k \cos \theta_{k_1}}{\mathcal{D}_I} \right\} \cos(\phi_1 - \phi_2) \right] \end{aligned} \quad (3.22)$$

with $\phi_{1,2}$ parameterizing the spin vectors $s_{1,2}$ in 4 dimensions as in (2.12). Using (3.22), (3.21) and the formulae in Appendix A of Ref. 16), the calculation is straightforward, and gives $\Delta_T d\sigma_{R(NS)}^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-1)}$, up to corrections that vanish as $\epsilon \rightarrow 0$.

Combining the above results, we obtain $\Delta_T d\sigma_{\bar{R}}^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-1)} = \Delta_T d\sigma_{R(S)}^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-1)} + \Delta_T d\sigma_{R(NS)}^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-1)}$. Then, using (3.1) with $p \rightarrow k_1$ and integrating over θ_{k_1} , we finally derive the ‘‘azimuthal’’ angular distribution of one of the leptons

for one-gluon radiation:

$$\begin{aligned}
\frac{\Delta_T d\sigma_R^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}} &= \frac{16\pi^{1-\epsilon}}{\Gamma(1-\epsilon)} G(\epsilon) Q^2 \left[\frac{1}{\epsilon^2} \delta(1-z) - \frac{1}{\epsilon} \frac{2}{(1-z)_+} \right] \\
&\times \left[2^{1-2\epsilon} B(2-\epsilon, 2-\epsilon) \alpha_\Phi + \epsilon 2^{-2\epsilon} B(1-\epsilon, 1-\epsilon) (s_1 \cdot s_2) \right] \\
&+ 16\pi G(0) Q^2 \frac{2}{3} \cos(2\phi_{k_1} - \phi_1 - \phi_2) \\
&\times \left[-\frac{\pi^2}{12} \delta(1-z) + 2 \left(\frac{\ln(1-z)}{1-z} \right)_+ + \frac{1-z}{z} - \frac{3}{2} \frac{\ln^2 z}{1-z} - \frac{\ln z}{1-z} \right].
\end{aligned} \tag{3.23}$$

Apparently, this final form is valid in any frame in which the momenta \mathbf{p}_1 and \mathbf{p}_2 of the initial quark and antiquark in (2.2) are collinear along the z -axis.

3.3. Partonic cross section to $\mathcal{O}(\alpha_s)$ in the $\overline{\text{MS}}$ factorization scheme

With the tree-plus-virtual contribution (3.6) added to (3.23), the double poles in ϵ cancel out, while the single-pole terms remain. The result gives the azimuthal angular distribution to $\mathcal{O}(\alpha_s)$ as

$$\frac{\Delta_T d\sigma^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}} = \frac{\Delta_T d\sigma_{T+V}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}} + \frac{\Delta_T d\sigma_R^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}} = \frac{\Delta_T d\sigma_T^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}} + \frac{\Delta_T d\sigma_{V+R}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}}, \tag{3.24}$$

for the parton-level process (2.2) in the dimensional regularization. Here, in the second equality, we have rearranged $\Delta_T d\sigma^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-2)}$ into a sum of the $\mathcal{O}(\alpha_s^0)$ and $\mathcal{O}(\alpha_s^1)$ contributions. The remaining single-pole terms in $\Delta_T d\sigma_{V+R}^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-2)}$ represent the mass singularity associated with the emission of the collinear gluon. Indeed, these single-pole terms are completely absorbed into the parton distributions as the corresponding mass singularities: We substitute the result (3.24) into the LHS of the factorization formula (2.3) applied to the parton-level process (2.2) and (A.3) in Appendix A into $\delta H(x_1, x_2; \mu_F^2)$ in the RHS as the transversity distributions in the $\overline{\text{MS}}$ scheme for the incoming quark and antiquark [see (2.4)]. Matching both sides of the resulting formula, we determine the partonic cross section $\Delta_T d\hat{\sigma}/dQ^2 d\Omega_{k_1}^{(D-2)}$ on the RHS order-by-order in α_s . The matching at LO shows that the $\mathcal{O}(\alpha_s^0)$ term of $\Delta_T d\hat{\sigma}/dQ^2 d\Omega_{k_1}^{(D-2)}$ reads

$$\begin{aligned}
\frac{\Delta_T d\hat{\sigma}^{(0)}(p_1, p_2; Q^2; \mu_F^2)}{dQ^2 d\Omega_{k_1}^{(D-2)}} &= \frac{1}{e_q^2} \frac{\Delta_T d\sigma_T^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}} \\
&= \frac{1}{16\pi^2 N_c} \left(\frac{4\pi\alpha}{Q^2} \right)^2 \left(\frac{Q^2}{16\pi^2} \right)^{-\epsilon} \delta(1-z) \\
&\times \left[2^{1-2\epsilon} B(2-\epsilon, 2-\epsilon) \alpha_\Phi + \epsilon 2^{-2\epsilon} B(1-\epsilon, 1-\epsilon) (s_1 \cdot s_2) \right],
\end{aligned} \tag{3.25}$$

exactly for arbitrary ϵ . Apparently the RHS of (3.25) is free from the $1/\epsilon$ singularities and does not depend on μ_F . Similarly, the matching at the NLO level yields the $\mathcal{O}(\alpha_s^1)$ term of $\Delta_T d\hat{\sigma}/dQ^2 d\Omega_{k_1}^{(D-2)}$ as

$$\begin{aligned} \frac{\Delta_T d\hat{\sigma}^{(1)}(p_1, p_2; Q^2; \mu_F^2)}{dQ^2 d\Omega_{k_1}^{(D-2)}} &= \frac{1}{e_q^2} \left. \frac{\Delta_T d\sigma_{V+R}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}} \right|_{\mu \rightarrow \mu_F} + \frac{1}{2\hat{\epsilon}} \frac{\alpha_s}{\pi} \int dx \Delta_T P_{qq}(x) \\ &\times \left(\frac{\Delta_T d\hat{\sigma}^{(0)}(xp_1, p_2; Q^2; \mu_F^2)}{dQ^2 d\Omega_{k_1}^{(D-2)}} + \frac{\Delta_T d\hat{\sigma}^{(0)}(p_1, xp_2; Q^2; \mu_F^2)}{dQ^2 d\Omega_{k_1}^{(D-2)}} \right), \end{aligned} \quad (3.26)$$

for the $\overline{\text{MS}}$ factorization of the mass singularities at the scale μ_F , where $1/\hat{\epsilon} = 1/\epsilon - \gamma_E + \ln(4\pi)$, with γ_E the Euler constant, and $\Delta_T P_{qq}(x)$ is the LO Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) splitting function for the transversity distributions, given by (A.4). The terms proportional to $1/\hat{\epsilon}$ in (3.26) are generated by the $\mathcal{O}(\alpha_s)$ term of the parton distributions (A.1), combined with the $\mathcal{O}(\alpha_s^0)$ terms of the partonic cross section, (3.25), and cancel the $1/\epsilon$ poles in the first term on the RHS of (3.26). As a result, (3.26) is also finite as $\epsilon \rightarrow 0$.

Now, the mass singularities for the parton-level process (2.2) have been completely factorized into the relevant parton distributions, and the $\epsilon \rightarrow 0$ limit of the sum of (3.25) and (3.26) gives the final result for the partonic cross section (2.5) to $\mathcal{O}(\alpha_s)$ in 4 dimensions,

$$\begin{aligned} \frac{\Delta_T d\hat{\sigma}(p_1, p_2; Q^2; \mu_F^2)}{dQ^2 d\phi_{k_1}} &= \frac{\alpha^2}{3N_c s Q^2} \cos(2\phi_{k_1} - \phi_1 - \phi_2) \\ &\times \left(\delta(1-z) + \frac{\alpha_s}{2\pi} \left[2 \Delta_T P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + C_F \left\{ 8z \left(\frac{\ln(1-z)}{1-z} \right)_+ \right. \right. \right. \\ &\left. \left. \left. - 6 \frac{z \ln^2 z}{1-z} - 4 \frac{z \ln z}{1-z} + 4(1-z) + \left(\frac{2\pi^2}{3} - 8 \right) \delta(1-z) \right\} \right] \right), \end{aligned} \quad (3.27)$$

where ϕ_{k_1} and $\phi_{1,2}$ are defined as in (2.10) and (2.12), μ_F is the $\overline{\text{MS}}$ factorization scale, and we have used the fact [see (3.4)]

$$\alpha_\Phi(D \rightarrow 4) = \cos(2\phi_{k_1} - \phi_1 - \phi_2).$$

The result (3.27), along with (A.4), coincides with that obtained in previous works employing the massive gluon scheme¹⁶⁾ and the dimensional reduction scheme,¹⁷⁾ via the scheme transformation relation¹⁹⁾ (see also Ref. 20)). Substituting (3.27), and also the NLO transversity distributions for the hadrons $h_{1,2}$ in the $\overline{\text{MS}}$ scheme, into the RHS of (2.3), we obtain the NLO QCD prediction for the spin-dependent cross section of tDY (2.1) in hadron-hadron collisions, $\Delta_T d\sigma/dQ^2 d\phi_{k_1}$, as the mass (Q) and the azimuthal angular (ϕ_{k_1}) distribution associated with the observed lepton pair.

§4. The Q_T differential cross section to $\mathcal{O}(\alpha_s)$

In this section, we extend the calculation of the last section to the case in which the final state in (2.1) is observed in more detail. We calculate the spin-dependent cross section for tDY, which is differential also in the transverse momentum Q_T and rapidity y of the produced lepton pair,

$$\frac{\Delta_T d\sigma}{dQ^2 dQ_T^2 dy d\phi_{k_1}}, \quad (4.1)$$

taking into account the QCD mechanism up to $\mathcal{O}(\alpha_s)$ in perturbation theory. The rapidity of the lepton pair is defined by

$$y = \frac{1}{2} \ln \frac{Q^0 + Q^3}{Q^0 - Q^3} \quad (4.2)$$

in the hadron-hadron CM system for (2.1), in which we work in this section. Here, Q^0 and Q^3 are the components of the dilepton's momentum, $Q^\mu = (Q^0, \mathbf{Q}_T, Q^3)$, with Q^3 denoting the component along the direction of the colliding beam, while Q_T is defined as $Q_T = |\mathbf{Q}_T|$. Now, in the factorization formula (2.3) for (4.1), the corresponding partonic cross section on the RHS depends also on Q_T and y , as $\Delta_T d\hat{\sigma}(p_1, p_2; Q_T^2, Q^2, y; \mu_F^2)/dQ^2 dQ_T^2 dy d\phi_{k_1}$. Similarly to the last section, this partonic cross section can be obtained by matching the LHS and RHS of the factorization formula (2.3) in the case of the parton-level process (2.2) with Q_T and y observed. The LHS including the $\mathcal{O}(\alpha_s)$ corrections is represented by the diagrams in Fig. 1, which we calculate using dimensional regularization and obtain $\Delta_T d\sigma^{q\bar{q}}/dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)} = \Delta_T d\sigma_{T+V+R}^{q\bar{q}}/dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}$ as the tree-plus-virtual contribution plus the real emission [compare with (3.24)].^{*)} As demonstrated in the last section, the corresponding order-by-order matching accomplishes the factorization of mass singularities in terms of $\delta H(x_1, x_2)$ of (2.4). Because $\delta H(x_1, x_2)$ is universal (process independent), the factorization of the mass singularities can be expressed by the relation between $\Delta_T d\sigma^{q\bar{q}}/dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}$ and $\Delta_T d\hat{\sigma}/dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}$, which is analogous to (3.25) and (3.26):

$$\begin{aligned} \frac{\Delta_T d\hat{\sigma}(p_1, p_2; Q_T^2, Q^2, y; \mu_F^2)}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} &= \frac{1}{e_q^2} \left[\left. \frac{\Delta_T d\sigma_{T+V+R}^{q\bar{q}}}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} \right|_{\mu \rightarrow \mu_F} + \frac{\Delta_T d\sigma_{CT}^{q\bar{q}}}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} \right] \\ &\equiv \frac{\Delta_T d\hat{\sigma}_{T+V+R+CT}(p_1, p_2; Q_T^2, Q^2, y; \mu_F^2)}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}}, \end{aligned} \quad (4.3)$$

^{*)} Here and below, $\Delta_T d\sigma_V^{q\bar{q}}/dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}$ denotes the contribution of the virtual-correction diagram in Fig. 1, combined with the contribution due to the wave-function renormalization factors for the incoming quark and antiquark legs [see the discussion below (2.15)].

where the second term on the RHS of the first line plays the role of the “counter-term” to cancel the mass singularities in the “V+R” contributions contained in the first term, and it reads, in the $\overline{\text{MS}}$ factorization scheme,

$$\begin{aligned} \frac{\Delta_T d\sigma_{\text{CT}}^{q\bar{q}}}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} &= \frac{1}{2\hat{\epsilon}} \frac{\alpha_s}{\pi} \int dx \Delta_T P_{qq}(x) \\ &\times \left[\left. \frac{\Delta_T d\sigma_{\text{T}}^{q\bar{q}}}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} \right|_{p_1 \rightarrow xp_1} + \left. \frac{\Delta_T d\sigma_{\text{T}}^{q\bar{q}}}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} \right|_{p_2 \rightarrow xp_2} \right], \end{aligned} \quad (4.4)$$

with the LO splitting function $\Delta_T P_{qq}(x)$ of (A.4). From (2.13), (2.15), and (3.2), it is straightforward to show

$$\frac{\Delta_T d\sigma_{\text{T}}^{q\bar{q}}}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} = \delta(Q_T^2) \delta\left(y - \frac{1}{2} \ln \frac{p_1^0 + p_1^3}{p_2^0 - p_2^3}\right) \frac{\Delta_T d\sigma_{\text{T}}^{q\bar{q}}}{dQ^2 d\Omega_{k_1}^{(D-2)}}, \quad (4.5)$$

exactly in arbitrary $D = 4 - 2\epsilon$ dimensions, where $\Delta_T d\sigma_{\text{T}}^{q\bar{q}}/dQ^2 d\Omega_{k_1}^{(D-2)}$ on the RHS is given by (3.25). At the tree level, apparently, it is impossible for the outgoing dilepton to have nonzero transverse momentum, and the rapidity is also determined completely by the kinematics of the initial state given by (2.2).

It is not difficult to identify the $1/\epsilon$ poles in the “V+R” contributions through a careful treatment of the relevant phase space integration in D dimensions, as done in (3.16)-(3.21) of the last section, and in so doing to verify that those pole contributions are indeed canceled out by the “CT” contribution in the partonic cross section (4.3). The result would yield the partonic cross section, which is finite as $D \rightarrow 4$ and is expressed in terms of the relevant partonic variables. However, rather than performing such manipulations directly at the partonic level, it is actually more convenient to perform the corresponding manipulations in the factorization formula (2.3) for the hadron-hadron collisions, with (4.3)-(4.5) substituted into the RHS as it is and their partonic variables re-expressed in terms of the relevant hadronic variables of (2.1). Because the finite value of the dilepton’s transverse momentum Q_T is a consequence of the recoil from the gluon radiation, the IR behavior associated with the real gluon emission is controlled by Q_T . Thus, rearranging the convolution integrals in the factorization formula (2.3) to make explicit its behavior as $Q_T \rightarrow 0$, we confirm the cancellation of the $1/\epsilon$ poles and simultaneously get the cross section formula for the hadron-hadron collisions, which is organized according to the Q_T dependence; such a form of the cross section is particularly suitable for the calculation of the dilepton Q_T spectrum in hadron-hadron collisions, and it is also even desirable when we attempt to include the soft gluon resummation effects relevant in the small- Q_T region in the next section.

We denote the total CM energy in (2.1) as \sqrt{S} , and thus we have $P_1 = \frac{1}{2}\sqrt{S}(1, \mathbf{0}, 1)$, $P_2 = \frac{1}{2}\sqrt{S}(1, \mathbf{0}, -1)$, where $\mathbf{0}$ denotes the $D-2$ dimensional null vector. We decompose the cross section (2.3) into four pieces, corresponding to the contributions on the RHS of (4.3), as

$$\Delta_T d\sigma \equiv \Delta_T d\sigma_{T+V+R+CT}, \quad (4.6)$$

and introduce the following useful hadronic variables according to the treatment of the unpolarized DY process in Ref. 31):

$$\begin{aligned} \tau &= \frac{Q^2}{S}, \quad x_1^0 = \sqrt{\tau} e^y, \quad x_2^0 = \sqrt{\tau} e^{-y}, \\ x_1^+ &= \frac{Q^0 + Q^3}{\sqrt{S}} = \left(\frac{Q^2 + Q_T^2}{S} \right)^{\frac{1}{2}} e^y, \quad x_2^+ = \frac{Q^0 - Q^3}{\sqrt{S}} = \left(\frac{Q^2 + Q_T^2}{S} \right)^{\frac{1}{2}} e^{-y}. \end{aligned} \quad (4.7)$$

Here, the definition (4.2) of y is used, and $x_{1,2}^0$ are the DY scaling variables, as usual. First, using (2.13), (2.16) and (3.9), the partonic cross section for the one-gluon radiation of (4.3) can be written for $p_i = x_i P_i$ as

$$\begin{aligned} \Delta_T d\hat{\sigma}_R(x_1 P_1, x_2 P_2) &= \frac{1}{e_q^2} \Delta_T d\sigma_R^{q\bar{q}}|_{p_i=x_i P_i} \\ &= \frac{1}{2N_c s} C_F \left(\frac{4\pi\alpha g\mu^\epsilon}{Q^2} \right)^2 \Delta_T \mathcal{R} d\Phi_3 \\ &= \frac{G^H(\epsilon)}{x_1 x_2 (Q_T^2)^\epsilon} \frac{\Delta_T \mathcal{R}}{16 Q^2 \mathcal{D}_T^{2-2\epsilon}} \delta((x_1 P_1 + x_2 P_2 - Q)^2) d\Omega_Q^{(D-2)} d\Omega_{k_1}^{(D-1)} dQ^2 dQ_T^2 dy, \end{aligned} \quad (4.8)$$

where, for simplicity, we have denoted the factorization scale as μ and have suppressed some arguments of the partonic cross section $\Delta_T d\hat{\sigma}_R$. We have also introduced the following shorthand notation:

$$\begin{aligned} G^H(\epsilon) &= \frac{Q^4}{8\pi^2 N_c S} \left(\frac{4\pi\alpha}{Q^2} \right)^2 \left(\frac{Q^2}{16\pi^2} \right)^{-\epsilon} \frac{\alpha_s}{\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{\pi^\epsilon}{4\pi}, \\ \mathcal{D}_T &= \frac{Q \cdot k_1}{|\mathbf{k}_1|} = Q^0 - Q^3 \cos \theta_{k_1} - Q_T \sin \theta_{k_1} \mathbf{n}_Q^{(D-2)} \cdot \mathbf{n}_{k_1}^{(D-2)}, \end{aligned} \quad (4.9)$$

with $Q^{0,3} = (Q^2 + Q_T^2)^{1/2}(e^y \pm e^{-y})/2$ from (4.2). The quantity \mathcal{D}_T is generated in (4.8) as a phase space factor from $d\Phi_3$, while the delta function $\delta((x_1 P_1 + x_2 P_2 - Q)^2)$ comes from the on-shell condition for the final-state gluon and is given by $\delta(s + \hat{t} + \hat{u} - Q^2)$ in terms of the partonic variables in (2.6) and (2.7). Using the hadronic variables (4.7), this delta function becomes

$$\delta((x_1 P_1 + x_2 P_2 - Q)^2) = \frac{1}{S} \delta(x_1 x_2 - x_1 x_2^+ - x_2 x_1^+ + \tau). \quad (4.10)$$

It is important to notice that, in general, a convolution integral like (2.3) with the delta function (4.10) can be split into the two symmetric integrals³¹⁾, ^{*})

$$\begin{aligned} & \int dx_1 dx_2 f(x_1, x_2) \delta(x_1 x_2 - x_1 x_2^+ - x_2 x_1^+ + \tau) \\ &= \int_{\sqrt{\tau_+} e^y}^1 dx_1 \frac{f(x_1, x_2^*)}{x_1 - x_1^+} + \int_{\sqrt{\tau_+} e^{-y}}^1 dx_2 \frac{f(x_1^*, x_2)}{x_2 - x_2^+}, \end{aligned} \quad (4.11)$$

where^{**)}

$$x_1^* = \frac{x_2 x_1^+ - x_1^0 x_2^0}{x_2 - x_2^+}, \quad x_2^* = \frac{x_1 x_2^+ - x_1^0 x_2^0}{x_1 - x_1^+}, \quad \sqrt{\tau_+} = \sqrt{\frac{Q_T^2}{S}} + \sqrt{\tau + \frac{Q_T^2}{S}}. \quad (4.12)$$

In $\Delta_T \mathcal{R}$ of (4.8), we first consider the singular part of (3.8), which now reads

$$\frac{\Delta_T \mathcal{R}_S}{16 Q^2} = \frac{Q^2}{Q_T^2} \left[\frac{Q^2}{4 \mathcal{D}_T^2} \alpha_\Phi \sin^2 \theta_{k_1} + \epsilon \frac{1}{2} (s_1 \cdot s_2) \right], \quad (4.13)$$

and this yields the following contribution to (4.8):

$$\begin{aligned} \frac{\Delta_T d\hat{\sigma}_{R(S)}(x_1 P_1, x_2 P_2)}{dQ^2 dQ_T^2 dy} &= G^H(\epsilon) \frac{1}{(Q_T^2)^{1+\epsilon}} \frac{\tau}{x_1 x_2} M_T(\epsilon, Q_T^2) \\ &\quad \times \delta(x_1 x_2 - x_1 x_2^+ - x_2 x_1^+ + \tau) d\Omega_Q^{(D-2)} d\Omega_{k_1}^{(D-1)}, \end{aligned} \quad (4.14)$$

where

$$M_T(\epsilon, Q_T^2) \equiv \frac{1}{\mathcal{D}_T^{4-2\epsilon}} \frac{Q^2}{4} \alpha_\Phi \sin^2 \theta_{k_1} + \frac{1}{\mathcal{D}_T^{2-2\epsilon}} \epsilon \frac{1}{2} (s_1 \cdot s_2). \quad (4.15)$$

Inserting (4.14) into (2.3) and using (4.11), we obtain the contribution to $\Delta_T d\sigma_R$ in (4.6), associated with the singular part (4.13),

$$\begin{aligned} \frac{\Delta_T d\sigma_{R(S)}}{dQ^2 dQ_T^2 dy} &= G^H(\epsilon) \frac{1}{(Q_T^2)^{1+\epsilon}} \left[\int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} \right. \\ &\quad \left. + \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+} \delta H(x_1^*, x_2) \frac{\tau}{x_1^* x_2} \right] M_T(\epsilon, Q_T^2) d\Omega_Q^{(D-2)} d\Omega_{k_1}^{(D-1)}, \end{aligned} \quad (4.16)$$

where we have suppressed the μ^2 dependence of $\delta H(x_1, x_2)$. We follow the procedure used for the unpolarized DY process in Ref. 31), in order to isolate the singular terms in the limit

^{*}) When $\sqrt{\tau_+} e^y \geq 1$ ($\sqrt{\tau_+} e^{-y} \geq 1$), the first (second) term in the RHS of (4.11) vanishes, because of the support property of the parton distributions contained in $f(x_1, x_2)$ [see (2.3)].

^{**)} Note that we have $x_{1,2}^+ = \sqrt{\tau_+} e^{\pm y} = x_{1,2}^* = x_{1,2}^0$, when $Q_T^2 = 0$.

$Q_T^2 \rightarrow 0$ of the above equation, corresponding to the radiation of the collinear gluon:

$$\begin{aligned}
& \frac{\Delta_T d\sigma_{R(S)}}{dQ^2 dQ_T^2 dy} \\
&= G^H(\epsilon) \frac{1}{(Q_T^2)^{1+\epsilon}} \left[\int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \left\{ \delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} - \delta H(x_1^0, x_2^0) \right\} \right. \\
&+ \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+} \left\{ \delta H(x_1^*, x_2) \frac{\tau}{x_1^* x_2} - \delta H(x_1^0, x_2^0) \right\} \\
&+ \left. \delta H(x_1^0, x_2^0) \ln \frac{(1-x_1^+)(1-x_2^+)S}{Q_T^2} \right] M_T(\epsilon, Q_T^2) d\Omega_Q^{(D-2)} d\Omega_{k_1}^{(D-1)}. \quad (4.17)
\end{aligned}$$

We can further isolate the poles in Q_T^2 by using the identities

$$\begin{aligned}
& \frac{Q^{2\epsilon}}{(Q_T^2)^{1+\epsilon}} = \frac{1}{(Q_T^2)_+} - \frac{1}{\epsilon} \delta(Q_T^2) + \mathcal{O}(\epsilon), \\
& \frac{Q^{2\epsilon}}{(Q_T^2)^{1+\epsilon}} \ln \frac{Q^2}{Q_T^2} = \left(\frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ + \frac{1}{\epsilon^2} \delta(Q_T^2) + \mathcal{O}(\epsilon), \quad (4.18)
\end{aligned}$$

where the “+” distributions that regulate the singularity at $Q_T^2 = 0$ are defined such that³²⁾

$$\int_0^{p_T^2} dQ_T^2 \left(\frac{\ln^n(Q^2/Q_T^2)}{Q_T^2} \right)_+ = -\frac{1}{n+1} \ln^{n+1}(Q^2/p_T^2). \quad (4.19)$$

Then, analogously to the situation discussed with regard to (3.18) – (3.21) for the case of the Q_T -integrated cross section, we find that only the two limiting cases for $M_T(\epsilon, Q_T^2)$,

$$M_T(\epsilon = 0, Q_T^2 \neq 0) \quad \text{and} \quad M_T(\epsilon \neq 0, Q_T^2 = 0),$$

are required in order to evaluate the $(D-3)$ -dimensional angular integration $d\Omega_Q^{(D-2)}$ in (4.17), up to corrections that vanish as $\epsilon \rightarrow 0$. In these two cases, the integration over the scattering angle θ_{k_1} of the lepton can be also performed easily. After straightforward calculations, we obtain the “azimuthal” angular distribution of the lepton for one-gluon radiation as [see (2.10), (2.12)]

$$\begin{aligned}
& \frac{\Delta_T d\sigma_{R(S)}}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} = \frac{2\pi}{Q^2} G^H(0) \cos(2\phi_{k_1} - \phi_1 - \phi_2) \frac{1}{3} \left(1 + \frac{2Q_T^2}{Q^2} \right) \\
& \times \left\{ \delta H(x_1^0, x_2^0) \left[\frac{1}{(Q_T^2)_+} \ln \frac{(1-x_1^+)(1-x_2^+)S}{Q^2} + \left(\frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ \right] \right. \\
& + \frac{1}{(Q_T^2)_+} \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \left[\delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} - \delta H(x_1^0, x_2^0) \right] \\
& + \left. \frac{1}{(Q_T^2)_+} \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+} \left[\delta H(x_1^*, x_2) \frac{\tau}{x_1^* x_2} - \delta H(x_1^0, x_2^0) \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{2\pi}{Q^2} G^H(\epsilon) \frac{\pi^{-\epsilon} 2^{-2\epsilon}}{\Gamma(1-\epsilon)} [2 B(2-\epsilon, 2-\epsilon) \alpha_\Phi + \epsilon B(1-\epsilon, 1-\epsilon)(s_1 \cdot s_2)] \delta(Q_T^2) \\
& \times \left\{ \delta H(x_1^0, x_2^0) \left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{(1-x_1^0)(1-x_2^0)S}{Q^2} \right] \right. \\
& \quad - \frac{1}{\epsilon} \left(\int_{x_1^0}^1 \frac{dx_1}{x_1 - x_1^0} \left[\delta H(x_1, x_2^0) \frac{x_0}{x_1} - \delta H(x_1^0, x_2^0) \right] \right. \\
& \quad \left. \left. + \int_{x_2^0}^1 \frac{dx_2}{x_2 - x_2^0} \left[\delta H(x_1^0, x_2) \frac{x_2^0}{x_2} - \delta H(x_1^0, x_2^0) \right] \right) \right\}. \tag{4.20}
\end{aligned}$$

It is straightforward to show that the partonic cross section for the virtual correction, $\Delta_T d\hat{\sigma}_V(p_1, p_2)/dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}$ of (4.3), is given by (4.5) multiplied by $(\alpha_s C_F/e_q^2 \pi) (4\pi\mu^2/Q^2)^\epsilon [1/\Gamma(1-\epsilon)][-1/\epsilon^2 - 3/(2\epsilon) - 4 + \pi^2/2 + \mathcal{O}(\epsilon)]$, which is the factor appearing in the last parentheses of (3.6). Substituting this into (2.3), the virtual-correction contribution in (4.6) reads

$$\begin{aligned}
& \frac{\Delta_T d\sigma_V}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{(D-2)}} \\
& = \frac{2\pi}{Q^2} G^H(\epsilon) \frac{\pi^{-\epsilon} 2^{-2\epsilon}}{\Gamma(1-\epsilon)} [2 B(2-\epsilon, 2-\epsilon) \alpha_\Phi + \epsilon B(1-\epsilon, 1-\epsilon)(s_1 \cdot s_2)] \delta(Q_T^2) \\
& \quad \times \delta H(x_1^0, x_2^0) \left[-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - 4 + \frac{\pi^2}{2} + \mathcal{O}(\epsilon) \right]. \tag{4.21}
\end{aligned}$$

Combining this result with (4.20) and using the identity

$$\int_{x^0}^1 dz \frac{f(z) - f(1)}{1-z} = \int_{x^0}^1 dz \frac{f(z)}{(1-z)_+} - f(1) \ln(1-x^0),$$

we get

$$\begin{aligned}
& \frac{\Delta_T d\sigma_{R(S)+V}}{dQ^2 dQ_T^2 dy d\Omega_{k_1}^{D-2}} = \frac{2\pi}{Q^2} G^H(0) \cos(2\phi_{k_1} - \phi_1 - \phi_2) \frac{1}{3} \left(1 + \frac{2Q_T^2}{Q^2} \right) \\
& \times \left\{ \delta H(x_1^0, x_2^0) \left[\frac{1}{(Q_T^2)_+} \ln \frac{(1-x_1^+)(1-x_2^+)S}{Q^2} + \left(\frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ \right] \right. \\
& \quad + \frac{1}{(Q_T^2)_+} \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \left[\delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} - \delta H(x_1^0, x_2^0) \right] \\
& \quad \left. + \frac{1}{(Q_T^2)_+} \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+} \left[\delta H(x_1^*, x_2) \frac{\tau}{x_1^* x_2} - \delta H(x_1^0, x_2^0) \right] \right\} \\
& + \frac{2\pi}{Q^2} G^H(\epsilon) \frac{\pi^{-\epsilon} 2^{-2\epsilon}}{\Gamma(1-\epsilon)} [2 B(2-\epsilon, 2-\epsilon) \alpha_\Phi + \epsilon B(1-\epsilon, 1-\epsilon)(s_1 \cdot s_2)] \delta(Q_T^2)
\end{aligned}$$

$$\begin{aligned}
& \times \left\{ \left(-4 + \frac{\pi^2}{2} \right) \delta H(x_1^0, x_2^0) \right. \\
& \left. - \frac{1}{2C_F\epsilon} \left[\int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H\left(\frac{x_1^0}{z}, x_2^0\right) + \int_{x_2^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H\left(x_1^0, \frac{x_2^0}{z}\right) \right] \right\}, \tag{4.22}
\end{aligned}$$

with $\Delta_T P_{qq}(x)$ of (A.4). We now derive the “CT” contribution of (4.6) in the $\overline{\text{MS}}$ scheme, using (4.3)-(4.5) in (2.3), and add the result to (4.22). We observe that the mass singularity poles in ϵ arising in (4.22) are completely canceled by the $1/\epsilon$ poles of the CT contributions, and taking the $\epsilon \rightarrow 0$ limit, we obtain in 4 dimensions

$$\begin{aligned}
\frac{\Delta_T d\sigma_{\text{R}(S)+\text{V}+\text{CT}}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} &= \frac{2\pi}{Q^2} G^H(0) \cos(2\phi_{k_1} - \phi_1 - \phi_2) \frac{1}{3} \left(1 + \frac{2Q_T^2}{Q^2} \right) \\
&\times \left\{ \delta H(x_1^0, x_2^0) \left[\frac{1}{(Q_T^2)_+} \ln \frac{(1-x_1^+)(1-x_2^+)S}{Q^2} + \left(\frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ \right] \right. \\
&\quad + \frac{1}{(Q_T^2)_+} \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \left[\delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} - \delta H(x_1^0, x_2^0) \right] \\
&\quad + \frac{1}{(Q_T^2)_+} \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+} \left[\delta H(x_1^*, x_2) \frac{\tau}{x_1^* x_2} - \delta H(x_1^0, x_2^0) \right] \\
&\quad + \delta(Q_T^2) \left(\left[-4 + \frac{\pi^2}{2} \right] \delta H(x_1^0, x_2^0) + \frac{1}{2C_F} \ln \frac{Q^2}{\mu^2} \right. \\
&\quad \left. \left. \times \left[\int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H\left(\frac{x_1^0}{z}, x_2^0\right) + \int_{x_2^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H\left(x_1^0, \frac{x_2^0}{z}\right) \right] \right] \right) \right\}. \tag{4.23}
\end{aligned}$$

To completely isolate the growth of the form $\sim 1/Q_T^2$ as Q_T^2 decreases from the third and fourth lines of (4.23), one more step is required. We rewrite the integral in the third line as

$$\begin{aligned}
& \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \left[\delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} - \delta H(x_1^0, x_2^0) \right] \\
&= \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \left[\delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} - \delta H(x_1^0, x_2^0) \right] \\
&\quad - \int_{x_1^0}^1 \frac{dx_1}{x_1 - x_1^0} \left[\delta H(x_1, x_2^0) \frac{x_1^0}{x_1} - \delta H(x_1^0, x_2^0) \right] \\
&\quad + \int_{x_1^0}^1 \frac{dx_1}{x_1 - x_1^0} \left[\delta H(x_1, x_2^0) \frac{x_1^0}{x_1} - \delta H(x_1^0, x_2^0) \right] \\
&= \int dx_1 \delta H_1 + \frac{1}{2C_F} \int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H\left(\frac{x_1^0}{z}, x_2^0\right) - \delta H(x_1^0, x_2^0) \left(\frac{3}{4} + \ln \frac{1-x_1^0}{x_1^0} \right), \tag{4.24}
\end{aligned}$$

introducing the following shorthand notation for the integral that vanishes for $Q_T = 0$:

$$\begin{aligned} \int dx_1 \delta H_1 \equiv & \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \left[\delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} - \delta H(x_1^0, x_2^0) \right] \\ & - \int_{x_1^0}^1 \frac{dx_1}{x_1 - x_1^0} \left[\delta H(x_1, x_2^0) \frac{x_1^0}{x_1} - \delta H(x_1^0, x_2^0) \right]. \end{aligned} \quad (4.25)$$

We also rewrite the integral in the fourth line in the same way, using

$$\begin{aligned} \int dx_2 \delta H_2 \equiv & \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+} \left[\delta H(x_1^*, x_2) \frac{\tau}{x_1^* x_2} - \delta H(x_1^0, x_2^0) \right] \\ & - \int_{x_2^0}^1 \frac{dx_2}{x_2 - x_2^0} \left[\delta H(x_1^0, x_2) \frac{x_2^0}{x_2} - \delta H(x_1^0, x_2^0) \right] \\ = & \int dx_1 \delta H_1 \Big|_{y \rightarrow -y}. \end{aligned} \quad (4.26)$$

In this way, (4.23) finally becomes

$$\begin{aligned} \frac{\Delta_T d\sigma_{R(S)+V+CT}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} = & \frac{2\pi}{Q^2} G^H(0) \cos(2\phi_{k_1} - \phi_1 - \phi_2) \frac{1}{6} \left(1 + \frac{2Q_T^2}{Q^2} \right) \\ & \times \left\{ \frac{2}{Q_T^2} \left(\int dx_1 \delta H_1 + \int dx_2 \delta H_2 \right) + 2 \delta H(x_1^0, x_2^0) \frac{1}{Q_T^2} \ln \frac{(1 - x_1^+)(1 - x_2^+)}{(1 - x_1^0)(1 - x_2^0)} \right. \\ & + \delta H(x_1^0, x_2^0) \left[2 \left(\frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ - \frac{3}{(Q_T^2)_+} + (-8 + \pi^2) \delta(Q_T^2) \right] \\ & + \left(\frac{1}{(Q_T^2)_+} + \delta(Q_T^2) \ln \frac{Q^2}{\mu^2} \right) \\ & \left. \times \frac{1}{C_F} \left[\int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H \left(\frac{x_1^0}{z}, x_2^0 \right) + \int_{x_2^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H \left(x_1^0, \frac{x_2^0}{z} \right) \right] \right\}. \end{aligned} \quad (4.27)$$

Setting $\epsilon = 0$, the contribution to (4.8) from the nonsingular part of $\Delta_T \mathcal{R}$, (3.8), is easily calculated as

$$\begin{aligned} \frac{\Delta_T d\hat{\sigma}(x_1 P_1, x_2 P_2)_{R(NS)}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} = & - \frac{2\pi}{Q^2} G^H(0) \cos(2\phi_{k_1} - \phi_1 - \phi_2) \frac{1}{Q_T^2} \ln \left(1 + \frac{Q_T^2}{Q^2} \right) \\ & \times \frac{\tau}{x_1 x_2} \delta(x_1 x_2 - x_1 x_2^+ - x_2 x_1^+ + \tau). \end{aligned}$$

Thus, the contribution to the hadronic cross section, $\Delta_T d\sigma_R$ in (4.6), is simply

$$\frac{\Delta_T d\sigma_{R(NS)}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} = - \frac{2\pi}{Q^2} G^H(0) \cos(2\phi_{k_1} - \phi_1 - \phi_2) \frac{1}{Q_T^2} \ln \left(1 + \frac{Q_T^2}{Q^2} \right)$$

$$\times \left\{ \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \delta H(x_1, x_2^*) \frac{\tau}{x_1 x_2^*} + \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+} \delta H(x_1^*, x_2) \frac{\tau}{x_1^* x_2} \right\}. \quad (4.28)$$

Adding (4.27), (4.28), and the tree-level contribution in 4 dimensions [see (4.3)-(4.6) and compare with (4.21)],

$$\frac{\Delta_T d\sigma_T}{dQ^2 dQ_T^2 dy d\phi_{k_1}} = \frac{Q^2}{16\pi^2 N_c S} \left(\frac{4\pi\alpha}{Q^2} \right)^2 \frac{1}{3} \cos(2\phi_{k_1} - \phi_1 - \phi_2) \delta H(x_1^0, x_2^0) \delta(Q_T^2),$$

and using

$$\frac{2\pi}{Q^2} G^H(0) = \frac{Q^2}{16\pi^2 N_c S} \left(\frac{4\pi\alpha}{Q^2} \right)^2 \frac{\alpha_s}{\pi} C_F,$$

we reach the final expression for the differential cross section (4.1) for the tDY (2.1) as (4.6), including the $\mathcal{O}(\alpha_s)$ QCD mechanism in the $\overline{\text{MS}}$ factorization scheme:

$$\frac{\Delta_T d\sigma}{dQ^2 dQ_T^2 dy d\phi_{k_1}} = \mathcal{N} \cos(2\phi_{k_1} - \phi_1 - \phi_2) [\Delta_T X(Q_T^2, Q^2, y) + \Delta_T Y(Q_T^2, Q^2, y)]. \quad (4.29)$$

Here, we have

$$\mathcal{N} = \frac{Q^2}{16\pi^2 N_c S} \left(\frac{4\pi\alpha}{Q^2} \right)^2 \frac{1}{3} = \frac{\alpha^2}{3 N_c S Q^2},$$

$\Delta_T X$ contains all terms that are singular as $Q_T^2 \rightarrow 0$, behaving as $Q_T^{-2} \times (\ln(Q^2/Q_T^2) \text{ or } 1)$ or $\delta(Q_T^2)$, and $\Delta_T Y$ is the remaining “finite” part (containing less singular terms). Writing $\Delta_T X$ as

$$\Delta_T X(Q_T^2, Q^2, y) = \Delta_T X^{(0)}(Q_T^2, Q^2, y) + \Delta_T X^{(1)}(Q_T^2, Q^2, y), \quad (4.30)$$

we have

$$\begin{aligned} \Delta_T X^{(0)}(Q_T^2, Q^2, y) &= \delta H(x_1^0, x_2^0; \mu_F^2) \delta(Q_T^2), \\ \Delta_T X^{(1)}(Q_T^2, Q^2, y) &= \frac{\alpha_s(\mu_R^2)}{2\pi} \left\{ C_F \left[2 \left(\frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ - \frac{3}{(Q_T^2)_+} + (\pi^2 - 8) \delta(Q_T^2) \right] \delta H(x_1^0, x_2^0; \mu_F^2) \right. \\ &\quad + \left(\frac{1}{(Q_T^2)_+} + \delta(Q_T^2) \ln \frac{Q^2}{\mu_F^2} \right) \\ &\quad \times \left[\int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H\left(\frac{x_1^0}{z}, x_2^0; \mu_F^2\right) + \int_{x_2^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H\left(x_1^0, \frac{x_2^0}{z}; \mu_F^2\right) \right] \Big\}, \end{aligned} \quad (4.32)$$

and

$$\begin{aligned}
\Delta_T Y(Q_T^2, Q^2, y) &= \frac{\alpha_s(\mu_R^2)}{\pi} C_F \\
&\times \left\{ \left[\frac{2}{Q^2} - \frac{3}{Q_T^2} \ln \left(1 + \frac{Q_T^2}{Q^2} \right) \right] \right. \\
&\times \left[\int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \delta H(x_1, x_2^*; \mu_F^2) \frac{\tau}{x_1 x_2^*} \right. \\
&\quad \left. \left. + \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+} \delta H(x_1^*, x_2; \mu_F^2) \frac{\tau}{x_1^* x_2} \right] \right. \\
&\left. + \frac{1}{Q_T^2} \left[\int dx_1 \delta H_1 + \int dx_2 \delta H_2 + \delta H(x_1^0, x_2^0; \mu_F^2) \ln \frac{(1 - x_1^+)(1 - x_2^+)}{(1 - x_1^0)(1 - x_2^0)} \right] \right\}, \quad (4.33)
\end{aligned}$$

where we have recovered the notation of (2.3) for the factorization scale with the replacement $\mu^2 \rightarrow \mu_F^2$, so that μ_F in (4.32) and (4.33) denotes the $\overline{\text{MS}}$ factorization scale. We have also made explicit the dependence of the QCD coupling constant $\alpha_s(\mu_R^2)$ on the corresponding $\overline{\text{MS}}$ renormalization scale μ_R . When both of the colliding hadrons in (2.1) are polarized along the x -axis, we have $\phi_1 = \phi_2 = 0$ [see (2.12)]. Note that this result is invariant under the replacement $y \rightarrow -y$, as it should be.

Equation (4.29) with (4.32) and (4.33) was first obtained in Ref. 23), but its derivation was not described in detail there. Here we give a complete derivation, explaining all the necessary techniques to deal with the complications in the D -dimensional calculation involving transverse degrees of freedom. We note that, integrating (4.29) over Q_T^2 , we obtain the cross section $\Delta_T d\sigma/dQ^2 dy d\phi_{k_1}$ in the $\overline{\text{MS}}$ scheme, and the result coincides with that found in Ref.20). In Appendix B, we also report the expression for $\Delta_T d\sigma/dQ^2 dQ_T^2 d\phi_{k_1}$, integrating (4.29) over y . When we integrate (4.29) over both Q_T^2 and y , the result coincides with the corresponding total cross section associated with the partonic cross section (3.27), which was calculated in the last section.

For $Q_T^2 > 0$, $\Delta_T X^{(0)}$ in (4.32) vanishes. Also, the terms proportional to $\delta(Q_T^2)$ in (4.32), including those associated with the $+$ distribution, $([\ln^n(Q^2/Q_T^2)]/Q_T^2)_+$, do not contribute. The cross section (4.29) in this case is of $\mathcal{O}(\alpha_s)$, and it gives the LO QCD prediction for the spin-dependent cross section of the tDY in the large Q_T region:

$$\begin{aligned}
&\frac{\Delta_T d\sigma^{\text{LO}}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} \\
&= \mathcal{N} \cos(2\phi_{k_1} - \phi_1 - \phi_2) \left[\Delta_T X^{(1)}(Q_T^2, Q^2, y)|_{Q_T^2 > 0} + \Delta_T Y(Q_T^2, Q^2, y) \right]. \quad (4.34)
\end{aligned}$$

§5. Transverse-momentum resummation: general formulation

The Q_T -differential cross section of the tDY including one-loop corrections, (4.29), contains the singular part $\Delta_T X$ of (4.30)-(4.32) that grows as $\sim \alpha_s \ln(Q^2/Q_T^2)/Q_T^2$ and $\sim \alpha_s/Q_T^2$ as $Q_T^2 \rightarrow 0$. The calculations in the last section indicate that these large contributions associated with the singularities at $Q_T^2 = 0$ are induced as the recoil effects from the emission of a soft and/or collinear gluon. Actually, corrections of this type appear at each order of the perturbative calculation and become very large for $Q_T \ll Q$. Also, in the higher-loop corrections, the radiation of the soft and/or collinear partons produces the $1/\epsilon$ poles due to the IR divergences. After the factorization of the collinear singularities is accomplished in terms of the parton distribution functions, the remaining IR divergences are found to cancel when all the diagrams at the same order in α_s are combined. The Kinoshita-Lee-Nauenberg (KLN) theorem¹⁵⁾ ensures that the corresponding cancellation of the IR divergences occurs between the real-emission and virtual-loop contributions, due to the relevant soft-collinear radiations. However, when the final dilepton of (2.1) is kinematically constrained to have a small transverse-momentum Q_T , the real emission of the accompanying radiation is strongly inhibited. In this case, the KLN cancellation of the corresponding IR divergences still occurs, but it is “incomplete”, because the phase space for the multiple real-emission in the final state is strongly restricted for small Q_T ; specifically, all $1/\epsilon$ poles still cancel (at the leading power in $1/Q$) but the remainder depends in a singular manner on the small parameter Q_T that constrains the phase space. These remnants of the incomplete cancellation at the boundary of phase space actually produce a series of enhanced logarithmic contributions,

$$\alpha_s^n \frac{\ln^m(Q^2/Q_T^2)}{Q_T^2}, \quad (m = 2n - 1, 2n - 2, \dots, 1, 0) \quad (5.1)$$

at n -th order in α_s . When only soft gluons are radiated and the sum of their transverse momenta balances Q_T , the contributions corresponding to $m = 2n - 1$, as well as the contributions with $m = 2n - 2, 2n - 3, \dots$, arise as the IR-finite piece. When collinear gluons are also radiated, contributions with $m = 2n - 2, 2n - 3, \dots$ arise.*) These so-called “recoil logarithms” make the fixed-order perturbation theory invalid for $Q_T \ll Q$, and have to be resummed to all orders in α_s in order to make a reliable prediction of the cross section at small Q_T .

The corresponding resummation, the “ Q_T -resummation,” can be treated on the basis of the general formulation of Collins, Soper and Sterman (CSS).³²⁾ After reviewing the CSS formalism, emphasizing its universal structure, we use²³⁾ it to perform all-order resummation

*) The higher-order contributions in (5.1) can be produced by the radiation of semi-hard gluons which have Q_T as net transverse momentum.

of the recoil logarithms for the tDY cross section, up to NLL accuracy, which corresponds to completely summing the first three towers of logarithms, i.e., $\alpha_s^n \ln^m(Q^2/Q_T^2)/Q_T^2$ with $m = 2n - 1, 2n - 2$, and $2n - 3$, for all n . We also discuss various kinds of elaborations of our NLL resummation formula beyond the original CSS form, on the basis of recent developments in the Q_T -resummation formalism.^{34), 35), 44), 45)} Combining the resulting NLL-resummed cross section with the leading-order (LO) cross section (4.34) in a consistent matching procedure, we obtain the “NLL+LO” cross section of the tDY, which has a uniform accuracy over the entire range of Q_T .

5.1. Collins-Soper-Sterman (CSS) resummation formalism

First, we explain the Q_T -resummation formalism in a general form, such that it is not restricted to the present case of the tDY (2.1). For this purpose, we consider the process

$$h_1(P_1) + h_2(P_2) \rightarrow F(Q^2, Q_T^2, y; \phi) + X, \quad (5.2)$$

where the collisions of the hadrons h_1 and h_2 with momenta P_1 and P_2 produce a system of non-strongly interacting final-state particles, F , carrying total momentum Q , total transverse momentum Q_T and rapidity y . The additional variable ϕ denotes the possible dependence on the kinematics of the final-state particles in F , such as individual transverse momenta. For the case of the tDY azimuthal angular distribution discussed in §4, we have $\phi \rightarrow \phi_{k_1}$, the azimuthal angle of the final lepton. In general, $h_{1,2}$ may be polarized hadrons, but here their spins are suppressed for simplicity.

The CSS resummation formalism yields the cross section for this process, which is applicable to the small Q_T region as well as the large Q_T region. The corresponding CSS formula possesses the following general structure, decomposed into two types of terms:

$$\frac{d\sigma^{(\text{CSS})}}{dQ^2 dQ_T^2 dy d\phi} = \frac{d\sigma^{(\text{res.})}}{dQ^2 dQ_T^2 dy d\phi} + \frac{d\sigma^{(\text{fin.})}}{dQ^2 dQ_T^2 dy d\phi}. \quad (5.3)$$

Here, both terms on the RHS are expressed as convolutions of the corresponding partonic cross sections and the parton distributions, which are formally similar to the RHS of (2.3), but the partonic cross sections participating in (5.3) are more sophisticated than those calculated in §4. The first term in (5.3), $d\sigma^{(\text{res.})}/dQ^2 dQ_T^2 dy d\phi$, can be evaluated by resumming the “singular” terms, like $\Delta_T X$ of (4.30)-(4.32), to all orders in α_s . In particular, $d\sigma^{(\text{res.})}/dQ^2 dQ_T^2 dy d\phi$ collects the logarithmic contributions (5.1), which dominate the cross section at small Q_T . The second term, $d\sigma^{(\text{fin.})}/dQ^2 dQ_T^2 dy d\phi$, is the “finite” component that is not associated with the logarithmically-enhanced contributions (5.1), and thus it can be computed with fixed-order perturbation theory. Formally, we can define the corresponding

finite component analogously to $\Delta_T Y$ in (4.29) and (4.33); i.e., $d\sigma^{(\text{fin.})}/dQ^2 dQ_T^2 dy d\phi$ is obtained as the contributions less singular than $1/Q_T^2$ or $\delta(Q_T^2)$ as $Q_T^2 \rightarrow 0$ among the terms in the cross section $d\sigma/dQ^2 dQ_T^2 dy d\phi$, which is calculated up to an appropriate order in α_s . In (5.3), the first term dominates the cross section $d\sigma^{(\text{CSS})}/dQ^2 dQ_T^2 dy d\phi$ for $Q_T \ll Q$, and thus the second term is negligible in this region. But the second term becomes important when $Q_T \sim Q$.

In the CSS resummation formalism, the resummation of the logarithmic contributions (5.1) is carried out with the “ b -space” resummation approach, introducing a 2-dimensional impact parameter \mathbf{b} that is the Fourier conjugate of the transverse momentum \mathbf{Q}_T . As noted above, multiple-gluon emission induces (5.1) as its IR-finite piece, and the corresponding contributions can be resummed most straightforwardly in the b space, to all orders in the perturbation theory.²⁶⁾ After the resummation, the b -space cross section is Fourier-transformed back to the Q_T space. In this way, the first term in (5.3) is obtained as³²⁾

$$\frac{d\sigma^{(\text{res.})}}{dQ^2 dQ_T^2 dy d\phi} = \frac{1}{4\pi} \int d^2b e^{i\mathbf{b} \cdot \mathbf{Q}_T} \mathcal{W}(b; Q, y, \phi) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \mathcal{W}(b; Q, y, \phi), \quad (5.4)$$

where $J_0(bQ_T)$ is a Bessel function, and the b -space representation $\mathcal{W}(b; Q, y, \phi)$ has the general structure [using the notation of Ref. 34) and the hadronic variables (4.7) also for the present case (5.2)]

$$\begin{aligned} \mathcal{W}(b; Q, y, \phi) &= \sum_{j,i,k} \int_0^1 dx_1 \int_0^1 dx_2 \frac{d\sigma_{(\text{LO})}^{j\bar{j}}(Q^2)}{d\phi} \delta(Q^2 - x_1 x_2 S) \delta\left(y - \frac{1}{2} \ln \frac{x_1}{x_2}\right) \\ &\quad \times (C_{ji} \otimes f_{i/h_1})\left(x_1, \frac{b_0^2}{b^2}\right) (C_{\bar{j}k} \otimes f_{k/h_2})\left(x_2, \frac{b_0^2}{b^2}\right) e^{\mathcal{S}_j(b,Q)} \\ &= \frac{1}{S} \sum_j \frac{d\sigma_{(\text{LO})}^{j\bar{j}}(Q^2)}{d\phi} W_j(b; Q, x_1^0, x_2^0), \end{aligned} \quad (5.5)$$

with

$$W_j(b; Q, x_1, x_2) = \sum_{i,k} e^{\mathcal{S}_j(b,Q)} (C_{ji} \otimes f_{i/h_1})\left(x_1, \frac{b_0^2}{b^2}\right) (C_{\bar{j}k} \otimes f_{k/h_2})\left(x_2, \frac{b_0^2}{b^2}\right). \quad (5.6)$$

Here, the subscripts j, i and k can be q, \bar{q} or g , including the flavor degrees of freedom, $\sigma_{(\text{LO})}^{j\bar{j}}(Q^2)$ is the lowest-order cross section (integrated over Q_T^2)^{*)} for the parton-level process $j + \bar{j} \rightarrow F(Q^2)$, and $b_0 = 2e^{-\gamma_E}$, with γ_E being the Euler constant, has a kinematical origin. The quantity C_{ji} is the coefficient function, and $f_{i/h}(x, \mu^2)$ is the unpolarized or

^{*)} For the spin-dependent cross section in tDY, $j = q$ is relevant, where $\sigma_{(\text{LO})}^{q\bar{q}}(Q^2)$ is the LO cross section for the $q\bar{q}$ annihilation process $q + \bar{q} \rightarrow l\bar{l}(Q^2)$, which is equal to $\Delta_T \sigma_{\text{T}}^{q\bar{q}}$ calculated in §3 [see (3.25), (3.27)].

polarized distribution function for a parton i inside the hadron h . The symbol \otimes denotes their convolution, defined by

$$(C_{ji} \otimes f_{i/h})(x, \mu^2) = \int_x^1 \frac{dz}{z} C_{ji}(z, \alpha_s(\mu^2)) f_{i/h}(x/z, \mu^2). \quad (5.7)$$

The large logarithmic corrections are resummed into the Sudakov form factor $e^{\mathcal{S}_j(b, Q)}$ in terms of the exponent

$$\mathcal{S}_j(b, Q) = - \int_{b_0^2/b^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} \left\{ A_j(\alpha_s(\kappa^2)) \ln \frac{Q^2}{\kappa^2} + B_j(\alpha_s(\kappa^2)) \right\}, \quad (5.8)$$

with the perturbatively calculable functions $A_j(\alpha_s)$ and $B_j(\alpha_s)$. Here, for $Q \gg 1/b$, the leading contribution from the $A_j(\alpha_s)$ term is enhanced in comparison with that from the $B_j(\alpha_s)$ term due to an explicit logarithm $\ln(Q^2/\kappa^2)$ in the integrand. Other large logarithms are also implicit in the integration over κ^2 for both terms. In the form factor $e^{\mathcal{S}_j(b, Q)}$, the $A_j(\alpha_s)$ term of (5.8) represents exponentiation of large logarithms due to soft radiation, while the $B_j(\alpha_s)$ term represents exponentiation of large logarithms due to flavor-conserving collinear radiations.⁴⁴⁾

The quantity $W_j(b; Q, x_1, x_2)$ in (5.6) depends on Q^2 only through the Sudakov exponent (5.8). This simple structure is a consequence of the fact that $W_j(b; Q, x_1, x_2)$ is obtained as the solution of the following evolution equation:³²⁾

$$\frac{\partial}{\partial \ln Q^2} W_j(b; Q, x_1, x_2) = - \left[\int_{b_0^2/b^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} A_j(\alpha_s(\kappa^2)) + B_j(\alpha_s(Q^2)) \right] W_j(b; Q, x_1, x_2). \quad (5.9)$$

Moreover, the large logarithm of $Q^2 b^2$, arising in the integral $\int_{b_0^2/b^2}^{Q^2} (d\kappa^2/\kappa^2) A_j(\alpha_s(\kappa^2))$ of (5.9) when $b \sim 1/Q_T \gg 1/Q$, is controlled by the renormalization group (RG) equations governed by certain anomalous dimensions. For $j = q$, which is relevant to the DY and vector boson production, the corresponding RG equations, as well as (5.9), have been derived on the basis of the factorization property of the quark form factor in QCD into “hard,” “soft,” and “jet” factors to all orders in α_s , and also, exploiting the renormalization property of those individual factors, defined as matrix elements of gauge-invariant operators (see Refs. 32) and 48)).*)

The solution of (5.9) is generally expressed as

$$W_j(b; Q, x_1, x_2) = e^{\mathcal{S}_j(b, Q)} W_j(b; b_0/b, x_1, x_2), \quad (5.10)$$

*) In this approach, A_q and B_q appearing on the RHS of (5.9) are related to another set of two evolution kernels, denoted as “ K ” and “ G ” in Ref. 32). K and G , respectively, obey the RG equations governed by the so-called cusp anomalous dimension. This relation allows us to confirm that A_q and B_q contain no $\ln Q^2 b^2$ contributions and indeed take the forms of power series in α_s .

using the Sudakov exponent $\mathcal{S}_j(b, Q)$ of (5.8). With the boundary value $W_j(b; b_0/b, x_1, x_2)$ specified, (5.10) determines the complete behavior of $W_j(b; Q, x_1, x_2)$. For this purpose, we note that when $Q_T \ll Q$, and for the relevant region $b \sim 1/Q_T$, large logarithms of $Q^2 b^2 \sim Q^2/Q_T^2$ indeed arise in the Sudakov factor of (5.10) via the integral of (5.8). On the other hand, the boundary value $W_j(b; b_0/b, x_1, x_2)$ depends explicitly on only one distance scale, namely b , and thus, for $b \ll 1/\Lambda_{\text{QCD}}$, its b dependence is calculable by the customary perturbation theory. Specifically, $W_j(b; b_0/b, x_1, x_2)$ is determined by the singular component of the fixed-order differential cross section, which has a form analogous to that of $\Delta_T X$ in (4.30)-(4.32) obtained in §4. Setting $Q = b_0/b$ and the factorization scale as $\mu_F = b_0/b$ in formulae like (4.30)-(4.32), the Fourier transformation of the corresponding singular component from the Q_T space to the b space yields a result with the structure

$$W_j(b; b_0/b, x_1, x_2) = \sum_{i,k} (C_{ji} \otimes f_{i/h_1}) \left(x_1, \frac{b_0^2}{b^2} \right) (C_{\bar{j}k} \otimes f_{k/h_2}) \left(x_2, \frac{b_0^2}{b^2} \right), \quad (5.11)$$

which is factorized into two parts, corresponding to the incoming hadrons, h_1 and h_2 . This reflects the collinear nature of radiative corrections associated with the singular terms in the relevant region, $Q_T \sim 1/b$; i.e., the cross section receives logarithms of $b\Lambda_{\text{QCD}}$ only from collinear radiation, which can be treated with the DGLAP evolution of the parton distributions $f_{i/h}(x, b_0^2/b^2)$ associated with each hadron, and C_{ji} of (5.11) represents the remaining perturbative corrections as a power series in α_s . Therefore, combining the evolution equation (5.9) with (5.11) obtained from fixed-order perturbation theory, the general formula (5.6) follows.³²⁾ This analysis also demonstrates that (5.6) is accurate in the region $b \ll 1/\Lambda_{\text{QCD}}$.

We express the expansions of the functions $A_j(\alpha_s)$ and $B_j(\alpha_s)$ in (5.8) and the coefficient function $C_{ji}(z, \alpha_s)$ in (5.6) in powers of α_s as

$$\begin{aligned} A_j(\alpha_s) &= \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n A_j^{(n)}, \\ B_j(\alpha_s) &= \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n B_j^{(n)}, \\ C_{ji}(z, \alpha_s) &= \delta_{ji} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n C_{ji}^{(n)}(z). \end{aligned} \quad (5.12)$$

The coefficient $A^{(1)}$ yields the LL resummation, $\{A^{(2)}, B^{(1)}, C^{(1)}\}$ give the NLL terms, $\{A^{(3)}, B^{(2)}, C^{(2)}\}$ give the NNLL contributions, and so on. In particular, among the coefficients $\{A^{(1)}, A^{(2)}, B^{(1)}, C^{(1)}\}$ that are necessary for the NLL resummation, $\{A^{(1)}, A^{(2)}, B^{(1)}\}$ are known to be independent of the final states (see Refs. 34), 44) and 35)). For the DY process, (5.2) with $F = l\bar{l}$, which we consider in this paper, and also for other processes,

such as W and Z boson production, these coefficients with $j = q$ are necessary, and they are given by

$$A_q^{(1)} = 2 C_F, \quad A_q^{(2)} = 2 C_F K, \quad B_q^{(1)} = -3 C_F, \quad (5.13)$$

with²⁹⁾

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_G - \frac{5}{9} N_f, \quad (5.14)$$

where $C_G = N_c$, and N_f is the number of QCD massless flavors. The result (5.13) can be derived directly by evaluating certain loop diagrams which represent the evolution kernel of (5.9).^{32), 48)} Apparently, for $A_q^{(2)}$, this requires a two-loop calculation. Nevertheless, its value is known to be independent of the process as well as the spin. In fact, we can confirm that this is indeed the case by using a relation²⁹⁾ between $A_q^{(2)}$ and the usual two-loop DGLAP kernels for the parton distribution functions.^{*)} The parton distributions in (5.6) obey the DGLAP equation,¹⁴⁾

$$\mu^2 \frac{\partial}{\partial \mu^2} f_{i/h}(x, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_j \int_x^1 \frac{dz}{z} \mathcal{P}_{ij} \left(\frac{x}{z}, \alpha_s(\mu^2) \right) f_{j/h}(z, \mu^2), \quad (5.15)$$

with the corresponding DGLAP kernel $\mathcal{P}_{ij}(z, \alpha_s(\mu^2))$ given by a power series in α_s :

$$\mathcal{P}_{ij}(z, \alpha_s) = \mathcal{P}_{ij}^{(0)}(z) + \frac{\alpha_s}{2\pi} \mathcal{P}_{ij}^{(1)}(z) + \dots \quad (5.16)$$

The behavior of $\mathcal{P}_{ij}(z, \alpha_s(\mu^2))$ for $z \rightarrow 1$ is dominated by soft gluon emissions, and it is diagonal in i, j . Indeed, the coefficients $A_q^{(1)}$ and $A_q^{(2)}$ are, respectively, related to the dominant large- z behavior $\propto 1/(1-z)_+$ in the one- and two-loop terms of (5.16) for emission of gluons from a quark:

$$\mathcal{P}_{qq}^{(0)}(z) + \frac{\alpha_s}{2\pi} \mathcal{P}_{qq}^{(1)}(z) \rightarrow \frac{2 C_F}{(1-z)_+} + \frac{\alpha_s}{2\pi} \frac{2 C_F K}{(1-z)_+}. \quad (5.17)$$

The large z behavior (5.17) is universal to all DGLAP kernels associated with the twist-2 quark distributions, i.e., to the DGLAP kernels for density distribution $q(x, \mu^2)$, helicity distribution $\Delta q(x, \mu^2)$, and transversity distribution $\delta q(x, \mu^2)$. We also note that the term $\propto \delta(1-z)$ in the one-loop kernel $\mathcal{P}_{qq}^{(0)}(z)$ is also universal, and its coefficient determines the value of $B_q^{(1)}$ in (5.13).³⁴⁾ The similar large- z behavior

$$\mathcal{P}_{gg}^{(0)}(z) + \frac{\alpha_s}{2\pi} \mathcal{P}_{gg}^{(1)}(z) \rightarrow \frac{2 C_G}{(1-z)_+} + \frac{\alpha_s}{2\pi} \frac{2 C_G K}{(1-z)_+}$$

^{*)} The coefficients $A_q^{(1)}$ and $A_q^{(2)}$ are given by the one- and two-loop terms of the universal cusp anomalous dimension, respectively. The cusp anomalous dimension plays a role⁷³⁾ in the evolution of the quark distribution functions for large x .

and the universal coefficient of the $\delta(1-z)$ term in the one-loop kernel $\mathcal{P}_{gg}^{(0)}(z)$ lead to⁴⁹⁾

$$A_g^{(1)} = 2C_G, \quad A_g^{(2)} = 2C_G K, \quad B_g^{(1)} = -4\pi\beta_0,$$

with K of (5.14). Here, β_0 is the first coefficient of the QCD β function, and is given by

$$\beta_0 = \frac{11C_G - 2N_f}{12\pi}. \quad (5.18)$$

Recently, the three-loop term $\mathcal{P}_{ij}^{(2)}(z)$ in (5.16) for the unpolarized parton distributions has been calculated, and $A_j^{(3)}$ has been extracted from its $z \rightarrow 1$ behavior.⁵⁰⁾ We note that $A_j^{(n)}$ ($n \geq 1$) and, thus, the entire function $A_j(\alpha_s)$ are actually process independent. As for the other coefficients in (5.12), $B_j^{(n)}$ ($n \geq 2$) and $C_{ji}^{(n)}$ ($n \geq 1$) depend on the process.³⁴⁾ General expressions for the coefficients $B_j^{(2)}$ and $C_{ji}^{(1)}$, including the process-dependent pieces, are derived in Ref. 34) in the $\overline{\text{MS}}$ scheme.^{*)} It is possible to transform the CSS resummation formula given in (5.4)-(5.6) into a new form, so that the process independence of the building block of the resummation formula is maximal (see Ref. 44)).

The logarithms of $Q^2 b^2$ contained in the Sudakov factor $e^{\mathcal{S}(b,Q)}$, which are large in the region $b \gg 1/Q$, can be made explicit. Here we have suppressed the subscript j of the Sudakov exponent for simplicity. These contributions can be organized within a systematic large logarithmic expansion according to consistent order counting, where $\alpha_s \ln(Q^2 b^2)$ is formally considered of order unity.^{35), 45)} Substituting the expansions (5.12) and the explicit form of the running coupling constant,

$$\alpha_s(\kappa^2) = \frac{1}{\beta_0 \ln(\kappa^2/\Lambda_{\text{QCD}}^2)} - \frac{\beta_1 \ln \ln(\kappa^2/\Lambda_{\text{QCD}}^2)}{\beta_0^3 \ln^2(\kappa^2/\Lambda_{\text{QCD}}^2)} + \dots, \quad (5.19)$$

with β_0 of (5.18) and

$$\beta_1 = \frac{17C_G^2 - 5C_G N_f - 3C_F N_f}{24\pi^2}, \quad (5.20)$$

we perform the κ^2 integral in (5.8) and organize the resulting exponent $\mathcal{S}(b, Q)$ in terms of

$$\lambda = \beta_0 \alpha_s(\mu_R^2) \ln \frac{Q^2 b^2}{b_0^2} \equiv \beta_0 \alpha_s(\mu_R^2) L, \quad (5.21)$$

where L plays the role of the large logarithmic expansion parameter in the b space. Considering as $\lambda \sim \mathcal{O}(1)$, $\mathcal{S}(b, Q)$ can be systematically expanded as

$$\mathcal{S}(b, Q) = \frac{1}{\alpha_s(\mu_R^2)} h^{(0)}(\lambda) + h^{(1)}(\lambda) + \sum_{n=2}^{\infty} \left(\frac{\alpha_s(\mu_R^2)}{2\pi} \right)^{n-1} h^{(n)}(\lambda), \quad (5.22)$$

^{*)} $A_q^{(n)}$ ($n = 1, 2, \dots$) and $B_q^{(1)}$ are independent of the factorization scheme, but $B_q^{(n)}$ ($n \geq 2$) and $C_{ij}^{(n)}(z)$ ($n \geq 1$) depend on the factorization scheme (see e.g. Ref. 35)).

where $h^{(n)}(\lambda)$ ($n = 0, 1, \dots$) are functions of λ , which vanish at $\lambda = 0$, as $h^{(n)}(0) = 0$. Then, it is straightforward to find

$$h^{(0)}(\lambda) = \frac{A^{(1)}}{2\pi\beta_0^2} [\lambda + \ln(1 - \lambda)], \quad (5.23)$$

$$h^{(1)}(\lambda) = \frac{A^{(1)}\beta_1}{2\pi\beta_0^3} \left[\frac{1}{2} \ln^2(1 - \lambda) + \frac{\lambda + \ln(1 - \lambda)}{1 - \lambda} \right] + \frac{B^{(1)}}{2\pi\beta_0} \ln(1 - \lambda) - \frac{1}{4\pi^2\beta_0^2} \left[A^{(2)} - 2\pi\beta_0 A^{(1)} \ln \frac{Q^2}{\mu_R^2} \right] \left[\frac{\lambda}{1 - \lambda} + \ln(1 - \lambda) \right], \quad (5.24)$$

and $h^{(2)}(\lambda)$ has a similar form, which can be found in, e.g., Ref. 35); $h^{(2)}(\lambda)$ involves $A^{(3)}$, $B^{(2)}$ and the third coefficient of the QCD β function, β_2 , in addition to the coefficients appearing in (5.24). From (5.22), one can see that the Sudakov factor $e^{\mathcal{S}(b,Q)}$ is generically the exponentiation of the logarithmic terms $\alpha_s^n L^m$ with $m \leq n + 1$ at each order n : The first term, $h^{(0)}/\alpha_s$, in (5.22) collects the LL terms, $\alpha_s^n L^{n+1}$ ($n \geq 1$), and the second term, $h^{(1)}(\lambda)$, collects the NLL terms, $\alpha_s^n L^n$ ($n \geq 1$). Also, the third term, $(\alpha_s/2\pi)h^{(2)}$, controls the NNLL contributions, $\alpha_s^{n+1} L^n$ ($n \geq 1$), and so forth. Note that the ratio of two successive terms in (5.22) is formally of $\mathcal{O}(\alpha_s)$, and (5.22) is organized as a systematic power series in the small expansion parameter $\alpha_s(\mu_R^2)$, similarly to the customary perturbative expansions.

5.2. Matching with the fixed-order α_s calculation

To bring the CSS resummation formula (5.4)-(5.6) into contact with the fixed-order calculation of the cross section, we formally expand $\mathcal{W}(b; Q, y, \phi)$ in (5.4) in terms of $\alpha_s(\mu_R^2)$, and perform, order-by-order, the Fourier transformation to Q_T space. Here we consider the expansion of $\mathcal{W}(b; Q, y, \phi)$ up to $\mathcal{O}(\alpha_s)$. Using (5.22), we can immediately expand the Sudakov factor $e^{\mathcal{S}(b,Q)}$ in terms of $\alpha_s(\mu_R^2)$ as

$$e^{\mathcal{S}(b,Q)} = 1 - \frac{\alpha_s(\mu_R^2)}{2\pi} \left(\frac{1}{2} A^{(1)} L^2 + B^{(1)} L \right) + \mathcal{O}(\alpha_s^2), \quad (5.25)$$

with $L = \ln(Q^2 b^2/b_0^2)$. Also, the b dependences of the coefficient functions C_{ji} and the parton distributions $f_{i/h}$ in (5.5) can be expanded perturbatively in terms of $\alpha_s(\mu_R^2)$; specifically, (5.12) and (5.15) yield

$$C_{ji}(z, \alpha_s(b_0^2/b^2)) = \delta_{ji} \delta(1 - z) + \frac{\alpha_s(\mu_R^2)}{2\pi} C_{ji}^{(1)}(z) + \mathcal{O}(\alpha_s^2),$$

$$f_{i/h}(x, b_0^2/b^2) = f_{i/h}(x, \mu_F^2) - \frac{\alpha_s(\mu_R^2)}{2\pi} \ln \frac{\mu_F^2 b^2}{b_0^2} \sum_j \int_x^1 \frac{dz}{z} \mathcal{P}_{ij}^{(0)} \left(\frac{x}{z} \right) f_{j/h}(z, \mu_F^2) + \mathcal{O}(\alpha_s^2). \quad (5.26)$$

Substituting these expansions into (5.4), we encounter an integration of the type

$$\mathcal{I}_n \equiv \int_0^\infty db \frac{b}{2} J_0(bQ_T) \left(\ln \frac{b^2 Q^2}{b_0^2} \right)^n. \quad (5.27)$$

This can be calculated by taking the $\eta \rightarrow 0$ limit of the similar Fourier transformation of $(b^2 Q^2/b_0^2)^\eta$ [see (6.14) and (C.3) below]. For the present case, we need the results

$$\mathcal{I}_0 = \delta(Q_T^2), \quad \mathcal{I}_1 = -\frac{1}{(Q_T^2)_+}, \quad \mathcal{I}_2 = -2 \left(\frac{\ln Q^2/Q_T^2}{Q_T^2} \right)_+, \quad (5.28)$$

where the $+$ distributions are defined as (4.19), and we get

$$\begin{aligned} \frac{d\sigma^{(\text{res.})}}{dQ^2 dQ_T^2 dy d\phi} &= \frac{1}{S} \sum_j \frac{d\sigma_{(\text{LO})}^{j\bar{j}}(Q^2)}{d\phi} \left(\delta(Q_T^2) f_{j/h_1}(x_1^0, \mu_F^2) f_{\bar{j}/h_2}(x_2^0, \mu_F^2) \right. \\ &\quad + \frac{\alpha_s(\mu_R^2)}{2\pi} \left\{ \left[A_j^{(1)} \left(\frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ + B_j^{(1)} \frac{1}{(Q_T^2)_+} \right] f_{j/h_1}(x_1^0, \mu_F^2) f_{\bar{j}/h_2}(x_2^0, \mu_F^2) \right. \\ &\quad + \delta(Q_T^2) \left[\sum_i (C_{ji}^{(1)} \otimes f_{i/h_1})(x_1^0, \mu_F^2) f_{\bar{j}/h_2}(x_2^0, \mu_F^2) \right. \\ &\quad \left. \left. + \sum_k f_{j/h_1}(x_1^0, \mu_F^2) (C_{\bar{j}k}^{(1)} \otimes f_{k/h_2})(x_2^0, \mu_F^2) \right] \right. \\ &\quad \left. + \left(\frac{1}{(Q_T^2)_+} + \delta(Q_T^2) \ln \frac{Q^2}{\mu_F^2} \right) \left[\sum_i (\mathcal{P}_{ji}^{(0)} \otimes f_{i/h_1})(x_1^0, \mu_F^2) f_{\bar{j}/h_2}(x_2^0, \mu_F^2) \right. \right. \\ &\quad \left. \left. + \sum_k f_{j/h_1}(x_1^0, \mu_F^2) (\mathcal{P}_{\bar{j}k}^{(0)} \otimes f_{k/h_2})(x_2^0, \mu_F^2) \right] \right\} \Bigg) + \mathcal{O}(\alpha_s^2). \end{aligned} \quad (5.29)$$

Here, we identify the structure characteristic of the singular component of the fixed-order cross section, like that observed in $\Delta_T X$ of (4.30)-(4.32). In particular, the matching of (5.29) with the corresponding fixed-order cross section completely determines the coefficients $A^{(1)}$, $B^{(1)}$, and $C^{(1)}$, and the LO cross section $\sigma_{(\text{LO})}^{j\bar{j}}(Q^2)$ for the parton-level process $j + \bar{j} \rightarrow F(Q^2)$; for example, for the case of the spin-dependent cross section of tDY, the matching with (4.30)-(4.32) gives results identical to (5.13) for $A_q^{(1)}$ and $B_q^{(1)}$, and also yields

$$C_{qq}^{(1)}(z) = C_{q\bar{q}}^{(1)}(z) = C_F \left(\frac{\pi^2}{2} - 4 \right) \delta(1-z), \quad C_{q\bar{q}}^{(1)}(z) = C_{\bar{q}q}^{(1)}(z) = 0 \quad (5.30)$$

in the $\overline{\text{MS}}$ scheme and

$$\frac{1}{S} \frac{d\sigma_{(\text{LO})}^{q\bar{q}}(Q^2)}{d\phi} = \frac{1}{S} \frac{d\sigma_{(\text{LO})}^{\bar{q}q}(Q^2)}{d\phi} = \mathcal{N} e_q^2 \cos(2\phi_{k_1} - \phi_1 - \phi_2), \quad (5.31)$$

while all other quantities associated with the gluon index $j = g$ vanish.

The expansion (5.29) can be extended to include higher orders in α_s . The corresponding expansion up to $\mathcal{O}(\alpha_s^2)$ was carried out for $Q_T > 0$ in unpolarized DY,³⁰⁾ and the resulting $\mathcal{O}(\alpha_s^2)$ term involves the coefficients $A^{(2)}$ and $B^{(2)*}$. The matching of that result with the corresponding fixed-order cross section confirms the universal value of $A^{(2)}$ in (5.13) and also determines $B^{(2)}$ for unpolarized DY [see also Ref. 34) for results in other unpolarized processes of the type (5.2)]. This fact also implies that when we use the resummed component (5.4) up to NLL accuracy, $d\sigma^{(\text{fin.})}/dQ^2 dQ_T^2 dy d\phi$ in (5.3) should be taken as the finite component of the fixed-order α_s cross section, and the sum of those two components gives the cross section $d\sigma^{(\text{CSS})}/dQ^2 dQ_T^2 dy d\phi$, which is exact up to $\mathcal{O}(\alpha_s)$ when expanded in powers of α_s . Similarly, when we use the resummed component up to NNLL accuracy, $d\sigma^{(\text{fin.})}/dQ^2 dQ_T^2 dy d\phi$ should be taken as the finite component of the fixed-order cross section to $\mathcal{O}(\alpha_s^2)$, so that we obtain $d\sigma^{(\text{CSS})}/dQ^2 dQ_T^2 dy d\phi$, which is exact up to $\mathcal{O}(\alpha_s^2)$. Generalizing these considerations, the second term, $d\sigma^{(\text{fin.})}/dQ^2 dQ_T^2 dy d\phi$, in (5.3) should be determined by

$$\frac{d\sigma^{(\text{fin.})}}{dQ^2 dQ_T^2 dy d\phi} = \frac{d\sigma}{dQ^2 dQ_T^2 dy d\phi} \Big|_{\text{fo}} - \frac{d\sigma^{(\text{res.})}}{dQ^2 dQ_T^2 dy d\phi} \Big|_{\text{fo}}, \quad (5.32)$$

where the notation $(\dots)|_{\text{fo}}$ represents the expansion of the quantity (\dots) in powers of $\alpha_s(\mu_R^2)$ up to a given fixed order; i.e., the first term on the RHS, $d\sigma/dQ^2 dQ_T^2 dy d\phi|_{\text{fo}}$, is the cross section that is computed by truncating the customary perturbative expansion at a given fixed order in $\alpha_s(\mu_R^2)$. As noted above, when we consider the resummation at the NLL level, $(\dots)|_{\text{fo}}$ is the expansion up to $\mathcal{O}(\alpha_s(\mu_R^2))$. In this case, the second term of (5.32), $d\sigma^{(\text{res.})}/dQ^2 dQ_T^2 dy d\phi|_{\text{fo}}$, is given by (5.29), where the coefficients $A^{(1)}$ and $B^{(1)}$ in (5.12) and $C^{(1)}$, $\sigma_{(\text{LO})}^{j\bar{j}}(Q^2)$, determined similarly to (5.30) and (5.31), are substituted. When we consider the resummation at the NNLL level, $(\dots)|_{\text{fo}}$ is the expansion up to $\mathcal{O}((\alpha_s(\mu_R^2))^2)$.

The matching procedure for the finite component, represented by (5.32), guarantees that $d\sigma^{(\text{CSS})}/dQ^2 dQ_T^2 dy d\phi$ in (5.3) retains the complete information of the perturbative calculation up to the specified fixed order and, at the same time, incorporates the resummation of the logarithmically enhanced contributions to all orders. The fixed-order truncation of $d\sigma^{(\text{CSS})}/dQ^2 dQ_T^2 dy d\phi$ exactly reproduces the fixed-order cross section obtained in the customary QCD perturbation theory. In this sense, using the matching of (5.32), the resummed and fixed-order components are consistently combined in $d\sigma^{(\text{CSS})}/dQ^2 dQ_T^2 dy d\phi$ of (5.3) without any double counting.

^{*)} The $\mathcal{O}(\alpha_s^2)$ term involving $C^{(2)}$ is proportional to $\delta(Q_T^2)$, similarly to the $\mathcal{O}(\alpha_s)$ term involving $C^{(1)}$ in (5.29), and it vanishes for $Q_T > 0$.

§6. Systematizing the b -space resummation formula

In the b integration in the resummed component (5.4), we can distinguish three regions:^{26),32)} (i) $0 \leq b \lesssim 1/Q$, (ii) $1/Q \lesssim b \ll 1/\Lambda_{\text{QCD}}$, and (iii) $b \gtrsim 1/\Lambda_{\text{QCD}}$. These are relevant to the kinematical regions $Q_T \gtrsim Q$, $\Lambda_{\text{QCD}} \ll Q_T \lesssim Q$, and $Q_T \lesssim \Lambda_{\text{QCD}}$, respectively. We now go into further detail concerning the behavior and the physical content of the resummation formula in each of these three regions.

6.1. Region (i): $0 \leq b \lesssim 1/Q$

In the region (i), $0 \leq b \lesssim 1/Q$, the resummation is irrelevant and can be truncated at low order in α_s . Thus here, the customary fixed-order perturbation theory can be used. The resummed component associated with this region should be given by (5.29) with $Q_T \gtrsim Q$ to good accuracy. Apparently, in general, if the functions A, B and C of (5.12) and the QCD β function are evaluated up to order α_s^n , then the corresponding resummed component is correct to order α_s^n .

6.2. Region (ii): $1/Q \lesssim b \ll 1/\Lambda_{\text{QCD}}$

In the region (ii), $1/Q \lesssim b \ll 1/\Lambda_{\text{QCD}}$, all-order resummed perturbation theory can be used [see the discussion below (5.10)]. In this region, L in (5.21) can be large, but still we have, taking $\mu_R \simeq Q$ as usual,

$$0 < \lambda \ll \beta_0 \alpha_s (\mu_R^2) \ln \frac{Q^2}{\Lambda_{\text{QCD}}^2} \sim 1, \quad (6.1)$$

so that the Sudakov factor $e^{\mathcal{S}(b,Q)}$ with (5.22)-(5.24) can be expanded in a power series in λ . In principle, the b dependence of the coefficient functions C_{ji} and the parton distributions $f_{i/h}$ in (5.5) can be expanded similarly in powers of λ (see §7).

It is instructive to apply the corresponding expansion to (5.4) and (5.5) at LL accuracy, which means that we retain only the first term with $h^{(0)}$ in (5.22) for the Sudakov factor $e^{\mathcal{S}(b,Q)}$, and also the terms at the same level in the other factors. The coefficient function $C^{(1)}$ in (5.12), as well as the scale dependence of the parton distributions $f_{i/h}$, appears at the same logarithmic level as $B^{(1)}$, i.e., at the NLL level, as seen in (5.29) (also compare (5.24) and (7.17) below). Thus, at the LL level, those effects can be ignored as $(C_{ji} \otimes f_{i/h})(x, b_0^2/b^2) \rightarrow f_{j/h}(x, \mu_F^2)$ in (5.5). As a result, the b dependence of $\mathcal{W}(b; Q, y, \phi)$ in the integrand of (5.4) remains only in the Sudakov factor at the LL level [see (5.23)],

$$e^{\mathcal{S}(b,Q)} = e^{h^{(0)}(\lambda)/\alpha_s} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{A^{(1)}}{2\pi\beta_0^2\alpha_s} \sum_{m=2}^{\infty} \frac{\lambda^m}{m} \right)^n$$

$$= \sum_{n=0}^{\infty} \frac{\alpha_s^n}{n!} (\varpi_n^{(0)} L^{2n} + \varpi_n^{(1)} L^{2n-1} + \varpi_n^{(2)} L^{2n-2} + \dots + \varpi_n^{(n-1)} L^{n+1}), \quad (6.2)$$

where $\alpha_s = \alpha_s(\mu_R^2)$. In the second line, (5.21) has been substituted, and the series are rearranged according to their powers in the large logarithm L for each order in α_s , with $\varpi_n^{(0)} = (-A^{(1)}/4\pi)^n$, and other expansion coefficients, $\varpi_n^{(1)}, \varpi_n^{(2)}, \dots$, which can be expressed similarly in terms of $A^{(1)}$ and β_0 . Using the expansion (6.2), the Fourier transformation relevant to (5.4) at the LL level is given by

$$\begin{aligned} \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{\mathcal{S}(b,Q)} &\equiv \mathcal{T}^{\text{LL}}(Q_T^2, Q^2) \\ &= \sum_{n=0}^{\infty} \int_0^\infty db \frac{b}{2} J_0(bQ_T) \frac{\alpha_s^n}{n!} (\varpi_n^{(0)} L^{2n} + \varpi_n^{(1)} L^{2n-1} + \dots), \end{aligned} \quad (6.3)$$

and it can be completely expressed in terms of \mathcal{I}_m ($m = 2n, 2n-1, \dots$) in (5.27). Generalizing (5.28) for an arbitrary integer n (≥ 0), we get [see (C.3)],

$$\begin{aligned} \mathcal{I}_n &= \frac{d^n}{d\eta^n} \left\{ \left[\delta(Q_T^2) - \sum_{m=0}^{\infty} \frac{\eta^{m+1}}{m!} \left(\frac{\ln^m(Q^2/Q_T^2)}{Q_T^2} \right)_+ \right] \exp \left[-2 \sum_{r=1}^{\infty} \frac{\zeta(2r+1)}{2r+1} \eta^{2r+1} \right] \right\} \Big|_{\eta=0} \\ &= \varrho_n \delta(Q_T^2) - n \left(\frac{\ln^{n-1}(Q^2/Q_T^2)}{Q_T^2} \right)_+ \\ &\quad + \frac{2}{3} \zeta(3) n(n-1)(n-2)(n-3) \left(\frac{\ln^{n-4}(Q^2/Q_T^2)}{Q_T^2} \right)_+ + \dots, \end{aligned} \quad (6.4)$$

where $\varrho_n \equiv (d^n/d\eta^n) \exp[-2 \sum_{r=1}^{\infty} \zeta(2r+1) \eta^{2r+1}/(2r+1)]|_{\eta=0}$, $\zeta(n)$ is the Riemann zeta-function, and the ellipses denote the terms involving $(\ln^m(Q^2/Q_T^2)/Q_T^2)_+$ with $m \leq n-6$. Thus, for $\Lambda_{\text{QCD}} \ll Q_T \lesssim Q$, corresponding to the region (ii) $1/Q \lesssim b \ll 1/\Lambda_{\text{QCD}}$, (6.3) yields

$$\mathcal{T}^{\text{LL}}(Q_T^2, Q^2) = \sum_{n=1}^{\infty} \frac{\alpha_s^n}{n!} \left[-2n \left(-\frac{A^{(1)}}{4\pi} \right)^n \frac{\ln^{2n-1}(Q^2/Q_T^2)}{Q_T^2} + \dots \right], \quad (6.5)$$

where the ellipses denote the terms involving $\ln^m(Q^2/Q_T^2)/Q_T^2$ with $m = 2n-2, 2n-3, \dots$. This result demonstrates that (5.4) indeed resums the towers of logarithms (5.1) to all orders in α_s . Summing up the leading tower in (6.5) over all n , we get

$$\mathcal{T}^{\text{LL}}(Q_T^2, Q^2) = \frac{A^{(1)}\alpha_s}{2\pi} \frac{\ln(Q^2/Q_T^2)}{Q_T^2} \exp \left[-\frac{A^{(1)}\alpha_s}{4\pi} \ln^2(Q^2/Q_T^2) \right] + \dots \quad (6.6)$$

Substituting $A^{(1)}$ of (5.13), it is seen that this result reproduces the Sudakov quark form factor in the so-called “double leading logarithmic approximation (DLLA).”²⁴⁾

It is worth noting that, introducing the Fourier transform [see (5.28)],

$$\nu(k_T) \equiv \int \frac{d^2b}{4\pi} e^{-i\mathbf{b}\cdot\mathbf{k}_T} \left[-\frac{A^{(1)}\alpha_s}{4\pi} L^2 \right] = \frac{A^{(1)}\alpha_s}{2\pi} \left(\frac{\ln(Q^2/k_T^2)}{k_T^2} \right)_+, \quad (6.7)$$

the contribution from the leading tower of logarithms in (6.3) can be expressed as

$$\begin{aligned} & \sum_{n=1}^{\infty} \frac{1}{n!} \int \frac{d^2b}{4\pi} e^{i\mathbf{b}\cdot\mathbf{Q}_T} \left[-\frac{A^{(1)}\alpha_s}{4\pi} L^2 \right]^n \\ &= \pi \sum_{n=1}^{\infty} \frac{1}{n!} \int d^2k_{1,T} \cdots d^2k_{n,T} \delta^{(2)} \left(\mathbf{Q}_T + \sum_{l=1}^n \mathbf{k}_{l,T} \right) \frac{\nu(k_{1,T})}{\pi} \cdots \frac{\nu(k_{n,T})}{\pi}. \end{aligned} \quad (6.8)$$

Here, $\nu(k_T)$ in (6.7) represents the probability distribution for the emission of one soft gluon with transverse momentum k_T in the final state, as $(d\sigma^{\text{1-gluon}}/dk_T^2)/\sigma^{\text{total}} = \nu(k_T)$, with σ^{total} the total cross section.²⁶⁾ Indeed, terms with the same structure as (6.7) appear in (4.32) and (5.29). Therefore, the n -th order term in (6.8) represents the probability for the emission of n soft gluons in the final state as the product of the probabilities for n independent gluons.²⁷⁾ Thus, the exponentiation of the LL term in the Sudakov factor $e^{\mathcal{S}(b,Q)}$ in fact sums up the multiple soft-gluon emission probabilities to all orders.

Note that in each term of the expansion on the RHS of (5.29), transverse-momentum conservation is ensured by the delta function, $\delta^{(2)}(\mathbf{Q}_T + \sum_{l=1}^n \mathbf{k}_{l,T})$, and thus \mathbf{Q}_T of the dilepton in the final state is provided by the recoil from the radiation of soft gluons. This automatic and proper treatment of transverse-momentum conservation is one of the most important features of the impact parameter b -space approach.²⁶⁾ In fact, transverse-momentum conservation is particularly important when we approach the small Q_T region. For example, the contribution for $n = 2$ in (6.8) gives, using (6.4),

$$\begin{aligned} & \frac{\pi}{2!} \int d^2k_{1,T} d^2k_{2,T} \delta^{(2)}(\mathbf{Q}_T + \mathbf{k}_{1,T} + \mathbf{k}_{2,T}) \frac{\nu(k_{1,T})}{\pi} \frac{\nu(k_{2,T})}{\pi} \\ &= \frac{1}{2} \left(\frac{A^{(1)}\alpha_s}{2\pi} \right)^2 \left[- \left(\frac{\ln^3(Q^2/Q_T^2)}{Q_T^2} \right)_+ + \frac{4\zeta(3)}{(Q_T^2)_+} \right], \end{aligned} \quad (6.9)$$

where the first term corresponds to the $n = 2$ term given in (6.5). Inspecting the integration over the transverse momenta, $k_{1,T}$ and $k_{2,T}$, on the LHS of (6.9), we confirm that the first term on the RHS comes from the “strongly-ordered” phase space as $\delta^{(2)}(\mathbf{Q}_T + \mathbf{k}_{1,T}) \theta(k_{1,T}^2 - k_{2,T}^2) + (1 \leftrightarrow 2)$. Then, the second term is the non-strongly-ordered contribution corresponding to the emission of soft gluons whose transverse momenta, $k_{1,T}$ and $k_{2,T}$ ($\gtrsim Q_T$), add vectorially to balance \mathbf{Q}_T . Although the second term is formally subleading in (6.9), contributions of this type, called “kinematic logarithms,”^{27),51)} dominate the cross section in the asymptotic

limit, $Q \gg Q_T \gg \Lambda_{\text{QCD}}$, where the DLLA Sudakov factor of (6.6) leads to suppression of the (formally) leading contributions. Here, the probability of having a parton-antiparton annihilation into a lepton pair with no emission of gluons of transverse momenta greater than a fixed value decreases asymptotically faster than any power of Q^2 , and events at small Q_T may be obtained asymptotically only by the emission of at least two gluons whose transverse momenta are not small and add to a small value of Q_T . All the non-leading “kinematic logarithms” are correctly taken into account by imposing transverse-momentum conservation as in (6.8) and (6.9), and this is naturally realized in the b -space resummation approach with (5.4).*) Also, apparently, transverse-momentum conservation with the b -space resummation approach is crucial in the case of smaller Q_T corresponding to the region (iii).

Calculations similar to those in (6.2)-(6.9) can be performed including the NLL terms and higher logarithmic terms. It is straightforward to show that the NLL terms associated with $h^{(1)}$ of the Sudakov exponent (5.22) give rise to a series of contributions $\alpha_s^n L^{2n-1}, \alpha_s^n L^{2n-2}, \dots$ to (6.3), and thus yield contributions $\alpha_s^n \ln^m(Q^2/Q_T^2)/Q_T^2$ with $m = 2n - 2, 2n - 3, \dots$ to (6.5); similarly, the NNLL terms give rise to the contributions $\alpha_s^n \ln^m(Q^2/Q_T^2)/Q_T^2$ with $m = 2n - 4, 2n - 5, \dots$ to (6.5). Therefore, the resummation formula (5.4) at the LL level corresponds to summing up only the first tower of logarithms, $\alpha_s^n \ln^{2n-1}(Q^2/Q_T^2)/Q_T^2$ exactly, while the resummation formula up to NLL accuracy allows us to fully sum up the first three towers, i.e., $\alpha_s^n \ln^m(Q^2/Q_T^2)/Q_T^2$ with $m = 2n - 1, 2n - 2$, and $2n - 3$.

The convergence of the resummed perturbation series for (5.4) in the Q_T space is governed by $\alpha_s \ln^2(Q^2/Q_T^2)$ rather than α_s , and when Q_T is small, $\alpha_s \ln^2(Q^2/Q_T^2)$ will be large even for $\alpha_s \ll 1$. As Q_T approaches Q , we have $\ln(Q^2/Q_T^2) \ll 1$, and the all-order resummed perturbation theory of (5.4) is smoothly extrapolated to the region (i).

6.3. Region (iii): $b \gtrsim 1/\Lambda_{\text{QCD}}$

Next we turn to the region (iii), $b \gtrsim 1/\Lambda_{\text{QCD}}$. In this region, λ in (5.21) can be as large as 1, and hence the Sudakov factor $e^{\mathcal{S}(b,Q)}$ cannot be expanded in terms of λ , and the functional form exponentiating (5.22)-(5.24) has to be used as it is. Moreover, in this long-distance region, nonperturbative effects become relevant, which could modify the resummation formula, (5.5) and (5.6). In fact, when b becomes extremely large, so that $\lambda \rightarrow 1$, the resummation formula in the form of (5.5) and (5.6) breaks down. In this case, the functions (5.23) and (5.24) in the Sudakov exponent (5.22) are singular at $\lambda = 1$, and this singular behavior is related to the presence of the Landau pole in the perturbative running coupling $\alpha_s(\kappa^2)$ in

*) There have been efforts to formulate a “ Q_T -space” resummation approach, which is basically organized according to the strong ordering of the phase space. Thus this approach requires a hard and sophisticated task of including effects of the non-leading “kinematic logarithms” (see Ref. 51)).

QCD.

Therefore, to properly define the b integration in the resummation formula (5.4) for the corresponding long-distance region, relevant to $Q_T \lesssim \Lambda_{\text{QCD}}$, a prescription to deal with this singularity is required. Such a prescription may be accompanied by the modification of (5.6) to complement the relevant nonperturbative effects. For this purpose, two types of approaches are commonly used in the literature.^{*)} One is the so-called b_* prescription, which was proposed by CSS.³²⁾ In this approach, $W_j(b; Q, x_1, x_2)$ in (5.6) is replaced as

$$W_j(b; Q, x_1, x_2) \rightarrow W_j(b_*; Q, x_1, x_2) F^{NP}(b; Q, x_1, x_2), \quad (6.10)$$

where

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{\text{lim}}^2}}, \quad b_{\text{lim}} \sim 0.5 \text{ GeV}^{-1}. \quad (6.11)$$

Through this replacement, $W_j(b; Q, x_1, x_2)$ in (5.6), which is accurate for $b \ll 1/\Lambda_{\text{QCD}}$, is smoothly extrapolated to the extremely large b region, while the singularity is avoided, since $b_* \leq b_{\text{lim}}$ in $W_j(b_*; Q, x_1, x_2)$. This corresponds to effectively taking into account the “freezing” of the running coupling in the long-distance region due to the nonperturbative effects. The function F^{NP} represents possible nonperturbative effects, which may be interpreted as the contributions associated with the intrinsic transverse momentum of partons inside the colliding hadrons. F^{NP} is normally taken as a smearing factor of Gaussian type in b .³²⁾ Thus, in the large b region of the integration over b in (5.4), $W_j(b_*; Q, x_1, x_2)$ approaches a constant, $W_j(b_{\text{lim}}; Q, x_1, x_2)$, and F^{NP} acts as a damping factor. Many different parameterizations of F^{NP} have been proposed (see Ref. 36) and references therein).

Another approach⁴⁵⁾ consists of an extension of the minimal prescription⁵³⁾ proposed in the context of the so-called threshold resummation. This approach allows us to avoid the Landau singularity in a purely perturbative framework: Decomposing the Bessel function in (5.4) into two Hankel functions as

$$J_0(bQ_T) = \frac{1}{2}[H_0^{(1)}(bQ_T) + H_0^{(2)}(bQ_T)], \quad (6.12)$$

we deform the b -integration contour associated with $H_0^{(1)}(bQ_T)$ and $H_0^{(2)}(bQ_T)$ into the upper and lower half planes in the complex b -space, respectively, and obtain the following two convergent integrals as $|b| \rightarrow \infty$:

$$\begin{aligned} \frac{d\sigma^{(\text{res.})}}{dQ^2 dQ_T^2 dy d\phi} &= \int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) \mathcal{W}(b; Q, y, \phi) \\ &\quad + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \mathcal{W}(b; Q, y, \phi). \end{aligned} \quad (6.13)$$

^{*)} See Ref. 52), for an attempt to incorporate the nonperturbative effects as power corrections to the RG equations that control the large logarithms arising in the integral on the RHS of (5.9).

The new contour \mathcal{C}_\pm is taken to be from 0 to b_c on the real axis, followed by the two branches, $b = b_c + e^{\pm i\theta}t$ with $t \in \{0, \infty\}$ and $0 < \theta < \pi/4$. The constant b_c here is arbitrary in the interval $0 \leq b_c < b_L$, where $b_L = (b_0/Q)e^{1/(2\beta_0\alpha_s(\mu_R^2))}$ corresponds to the position of the singularity on the real axis as the solution of $\lambda = 1$. Equation (6.13) provides us with a (formally) consistent definition of a finite b -integral for the resummation formula within a perturbative framework with which, unlike in the above case of the b_* -prescription, no extra cut-off parameter b_{lim} is required in this contour deformation prescription. In this paper, we employ (6.13) to deal with the Landau singularity. (Also, in this case, possible nonperturbative effects in the large b region can be included, as seen in (9.3) below.^{*)})

We note that the contour deformation used to obtain (6.13) can be performed “safely” order-by-order in $\alpha_s(\mu_R^2)$, by expanding $\mathcal{W}(b; Q, y, \phi)$ in the integrand in powers of α_s . The above discussion concerning the regions (i) and (ii) implies that for (5.4) in the region $Q_T \gg \Lambda_{\text{QCD}}$ we can define $\mathcal{W}(b; Q, y, \phi)$ in the integrand as a power series in $\alpha_s(\mu_R^2)$, which can be organized similarly as the integrand of (6.3). In this case, (5.4) can be completely expressed by \mathcal{I}_n in (5.27), whose b integration is well-defined and yields distributions like (5.28) and (6.4). It is straightforward to show that we can perform the above contour deformation into \mathcal{C}_\pm with (6.12) for the b integration in \mathcal{I}_n ; in fact, on the LHS of the relation

$$\int_0^\infty db \frac{b}{2} J_0(bQ_T) \left(\frac{b^2 Q^2}{b_0^2} \right)^\eta = \sum_{m=0}^\infty \frac{\eta^m}{m!} \mathcal{I}_m, \quad (6.14)$$

the corresponding contour deformation can be performed using Cauchy’s theorem; i.e., the contributions to the integral from the contour in the limits $b \rightarrow 0$ and $|b| \rightarrow \infty$ are found to vanish using an appropriate analytic continuation for η . After such a contour deformation is performed for each term in the series of (5.4), the corresponding series can be summed up under the integrand of the deformed b -integration. This yields $\mathcal{W}(b; Q, y, \phi)$, with the Sudakov factor exponentiating (5.22). This final form coincides with (6.13) exactly, and is applicable to the region $Q_T \lesssim \Lambda_{\text{QCD}}$ as well as the region $Q_T \gg \Lambda_{\text{QCD}}$. Therefore, the choice of contours in (6.13) is equivalent to the original contour in (5.4), order-by-order in $\alpha_s(\mu_R^2)$, and also extends the applicability of the formula even into the low Q_T region. This also implies that all the above results obtained for the regions (i) and (ii), as well as those obtained in §5, are unchanged using the new form (6.13). Note that this is a consequence of the fact that (6.13) does not involve any cut-off parameter, like b_{lim} in (6.11), and it is an advantage of our approach over the b_* prescription.

Now we can use (6.13) as the precise form of the resummation formula to investigate its

^{*)} Actually, we have chosen $0 < \theta < \pi/4$ in the above contour \mathcal{C}_\pm , anticipating the Gaussian smearing factor (9.3) as the corresponding nonperturbative effects.

behavior for $Q_T \lesssim \Lambda_{\text{QCD}}$. Here we mention in particular a remarkable point regarding the case $Q_T \approx 0$: In this case, the behavior of (6.13) is controlled by a saddle point in the b integration. As shown in §8, the corresponding saddle point is on the real- b axis with $\lambda \sim 1$. The important role of the saddle point for $Q_T \approx 0$ was pointed out in Refs. 26), 28) and 32), using the “old” form, (5.4). Around the saddle point, we have $L \sim 1/\alpha_s(\mu_R^2) \sim \ln(Q^2/\Lambda_{\text{QCD}}^2)$. Because this implies that all logarithms, L and $\ln(Q^2/\Lambda_{\text{QCD}}^2) \sim 1/\alpha_s$, are regarded as equally large for $Q \gg \Lambda_{\text{QCD}}$, the resulting contributions to the resummation formula (6.13) are organized in terms of the single small parameter α_s , but with a classification of the contributions with respect to the order of α_s that differs from that of the customary perturbation theory that can be used in the region (i), corresponding to $Q_T \gtrsim Q$. In the Sudakov exponent (5.8), one such logarithm, $\ln(Q^2/\kappa^2)$, explicitly appears, and another large logarithm is implicit in the integration over κ^2 . Suppose that one wants to evaluate the resummation formula (6.13) in an approximation of “degree N ,”³²⁾ meaning that any corrections are suppressed by a factor of $[\ln(Q^2/\Lambda_{\text{QCD}}^2)]^{-(N+1)}$. Then, since two large logarithms multiply the function A in (5.8), one must evaluate A in (5.12) to order α_s^{N+2} . Similarly, one needs B to order α_s^{N+1} , C to order α_s^N , and, for the running of α_s , the β function to order α_s^{N+2} . In particular, if one wants an approximate result for (6.13) that will converge to the exact $Q_T \approx 0$ cross section as $Q \rightarrow \infty$, one needs a degree 0 approximation: A to order α_s^2 , B to order α_s^1 , C to order α_s^0 , and β to order α_s^2 .³²⁾ Note that this classification controlling (6.13) for $Q_T \approx 0$ is different from that in the customary perturbation theory for $Q_T \gtrsim Q$, and also from that in the all-order resummed perturbation theory, which controls the towers of logarithms (5.1) in the kinematical region, $\Lambda_{\text{QCD}} \ll Q_T \lesssim Q$. This point is discussed in detail in §8.

Combining the above considerations for the three regions in the resummation formula (6.13) with (5.5) and (5.6), we see that this formula represents a complicated object involving contributions of various orders and from various distance scales, due to the ubiquitous gluons. The impact parameter b -space formulation allows us to organize these complicated contributions into the universal form of (5.5), and this form directly embodies a simple physical picture behind the process (5.2): When the transverse momentum Q_T of the final-state system F is small, the emission of the accompanying radiation is strongly inhibited, so that only soft and collinear partons, which have low transverse momenta k_T , are radiated into the final state. The quantity $d\sigma_{(\text{LO})}^{j\bar{j}}(Q^2)/d\phi$ represents the hard annihilation of partons, and the associated virtual corrections at transverse-momentum scales $k_T \sim Q$ are included in the coefficient functions C_{ji} . The form factor $e^{\mathcal{S}_j(b,Q)}$ contains real and virtual contributions due to soft [the function $A_j(\alpha_s)$ in (5.8)] and flavor-conserving collinear [the function $B_j(\alpha_s)$ in (5.8)] radiation at scales $Q \gtrsim k_T \gtrsim 1/b$. At very low momentum scales, $k_T \lesssim 1/b$, real and virtual soft-gluon corrections cancel, because the cross section is IR safe, and only real

and virtual contributions due to collinear radiation remain, inducing the contributions to the coefficient functions $C_{ji}(z, \alpha_s(b_0^2/b^2))$ and the RG evolution of the parton distributions $f_{i/h}(x, b_0^2/b^2)$.

§7. The NLL+LO cross section for the tDY

The Q_T -differential cross section (5.3) is given by the sum of the resummed component (6.13) and the finite component determined by (5.32). For $Q_T \ll Q$, (5.3) is dominated by the resummed part, $d\sigma^{(\text{res.})}/dQ^2 dQ_T^2 dy d\phi$, which behaves as $\sim 1/Q_T^2$ [see (5.29), (6.5) and (6.6)]. On the other hand, for $Q_T \sim Q$, it approaches the fixed-order result, $d\sigma/dQ^2 dQ_T^2 dy d\phi|_{\text{fo}}$, since there is no logarithmic enhancement and $d\sigma^{(\text{res.})}/dQ^2 dQ_T^2 dy d\phi$ and $d\sigma^{(\text{res.})}/dQ^2 dQ_T^2 dy d\phi|_{\text{fo}}$ almost cancel.^{*)} Therefore, in (5.3) with (5.32), the resummed cross section, which is dominant at small Q_T , and the fixed-order cross section controlling the large Q_T region are most naturally matched at intermediate Q_T , without double counting. For example, as discussed in §5.2, the resummed component $d\sigma^{(\text{res.})}/dQ^2 dQ_T^2 dy d\phi$ at NLL accuracy is exact up to $\mathcal{O}(\alpha_s(\mu_R^2))$ when expanded in powers of $\alpha_s(\mu_R^2)$. Therefore, the NLL resummed cross section should be matched with the fixed-order cross section at LO, which is of order α_s for $Q_T > 0$ [see the discussion below (5.31)]. We refer to the resulting cross section (5.3) as the “NLL+LO” cross section, $d\sigma^{\text{NLL+LO}}/dQ^2 dQ_T^2 dy d\phi$. Similarly, the NNLL resummed cross section is matched with the NLO cross section, giving the result for the “NNLL+NLO” cross section, and so forth. Note also that, in order to evaluate the NLL+LO cross section with consistent accuracy, the NLO parton distributions have to be used as the input for $f_{i/h}(x, \mu^2)$. The evaluation of the NNLL+NLO cross section requires the NNLO parton distributions.

7.1. The spin-dependent cross section

In what follows, we calculate the NLL+LO spin-dependent cross section of the tDY, performing the matching with the fixed-order result (4.29) obtained in §4. We also “reorganize” the resummed component given in (6.13) and (5.5) beyond the CSS form. This is necessary for the consistent evaluation of the b integral in (6.13) to the required accuracy.

Following the general formula described in §§5, 6, we perform the Q_T resummation for tDY at the NLL level. At this level, we must know the coefficients $\{A^{(1)}, A^{(2)}, B^{(1)}, C^{(1)}\}$ in (5.12). Then, substituting (5.13), (5.30) and (5.31) into (6.13) and (5.5), the NLL-level resummed component of the cross section is given by $\mathcal{N} \cos(2\phi_{k_1} - \phi_1 - \phi_2) \Delta_T X^{\text{NLL}}$ [see

^{*)} This cancellation is often very subtle in a numerical sense. See, for example, Ref.54).

(4.29)], with

$$\begin{aligned} \Delta_T X^{\text{NLL}}(Q_T^2, Q^2, y) = & \int_C db \frac{b}{2} J_0(bQ_T) e^{S(b, Q)} \int_{x_1^0}^1 \frac{dz}{z} \int_{x_2^0}^1 \frac{dz'}{z'} \Delta_T C(z, \alpha_s(b_0^2/b^2)) \\ & \times \Delta_T C(z', \alpha_s(b_0^2/b^2)) \delta H\left(\frac{x_1^0}{z}, \frac{x_2^0}{z'}, \frac{b_0^2}{b^2}\right). \end{aligned} \quad (7.1)$$

Here, the relevant coefficient function is given by

$$\Delta_T C(z, \alpha_s) = \delta(1-z) + \frac{\alpha_s}{2\pi} \Delta_T C^{(1)}(z), \quad (7.2)$$

with

$$\Delta_T C^{(1)}(z) = C_F \left(\frac{\pi^2}{2} - 4 \right) \delta(1-z), \quad (7.3)$$

and

$$S(b, Q) = \frac{1}{\alpha_s(\mu_R^2)} h^{(0)}(\lambda) + h^{(1)}(\lambda) \quad (7.4)$$

with (5.23), (5.24) and (5.13) is the exponent (5.22) for the Sudakov quark form factor up to NLL accuracy. Above, we have also introduced the shorthand notation for the contour integration of (6.13):

$$\begin{aligned} \int_C db \frac{b}{2} J_0(bQ_T) \mathcal{W}(b; Q, y, \phi) \equiv & \int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) \mathcal{W}(b; Q, y, \phi) \\ & + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \mathcal{W}(b; Q, y, \phi). \end{aligned} \quad (7.5)$$

In (7.1), the parton distributions, as well as the coupling constant $\alpha_s(b_0^2/b^2)$ associated with the coefficient function $\Delta_T C^{(1)}$, depend on b , in addition to the Sudakov factor $e^{S(b, Q)}$. Similarly to the Sudakov exponent $S(b, Q)$ given in (7.4), we can reorganize this b dependence in terms of a systematic large logarithmic expansion, using λ of (5.21). In order to extract this b dependence explicitly, we take the double Mellin moments of (7.1) with respect to the DY scaling variables $x_{1,2}^0$ at fixed Q ,

$$\begin{aligned} \Delta_T X_{N_1, N_2}^{\text{NLL}}(Q^2, Q_T^2) \equiv & \int_0^1 dx_1^0 (x_1^0)^{N_1-1} \int_0^1 dx_2^0 (x_2^0)^{N_2-1} \Delta_T X^{\text{NLL}}(Q_T^2, Q^2, y) \\ = & \int_C db \frac{b}{2} J_0(bQ_T) e^{S(b, Q)} \Delta_T C_{N_1}(\alpha_s(b_0^2/b^2)) \Delta_T C_{N_2}(\alpha_s(b_0^2/b^2)) \\ & \times \delta H_{N_1, N_2}(b_0^2/b^2), \end{aligned} \quad (7.6)$$

where [see (2.4)]

$$\delta H_{N_1, N_2}(\mu^2) = \int_0^1 dx_1 x_1^{N_1-1} \int_0^1 dx_2 x_2^{N_2-1} \delta H(x_1, x_2; \mu^2)$$

$$\begin{aligned}
&= \sum_{j=q,\bar{q}} e_j^2 f_{j/h_1, N_1}(\mu^2) f_{\bar{j}/h_2, N_2}(\mu^2), \\
f_{j/h, N}(\mu^2) &\equiv \int_0^1 dx x^{N-1} f_{j/h}(x, \mu^2),
\end{aligned} \tag{7.7}$$

with $f_{q/h}(x, \mu^2) \equiv \delta q_h(x, \mu^2)$, $f_{\bar{q}/h}(x, \mu^2) \equiv \delta \bar{q}_h(x, \mu^2)$, and^{*)}

$$\Delta_T C_N(\alpha_s) \equiv \int_0^1 dx x^{N-1} \Delta_T C(x, \alpha_s) = 1 + \frac{\alpha_s}{2\pi} \Delta_T C_N^{(1)}. \tag{7.8}$$

We rewrite the moment of the coefficient function, $\Delta_T C_N(\alpha_s(b_0^2/b^2))$, in (7.6) formally as

$$\Delta_T C_N(\alpha_s(b_0^2/b^2)) = \Delta_T C_N(\alpha_s(Q^2)) \exp \left\{ \ln \frac{\Delta_T C_N(\alpha_s(b_0^2/b^2))}{\Delta_T C_N(\alpha_s(Q^2))} \right\}. \tag{7.9}$$

Also, note that we obtain the following large logarithmic expansion of the running coupling constant (5.19) using (5.21):⁴⁵⁾

$$\begin{aligned}
&\ln \left(\frac{\alpha_s(Q^2)}{\alpha_s(b_0^2/b^2)} \right) \\
&= \ln(1 - \lambda) + \alpha_s(\mu_R^2) \left[\frac{\beta_1 \ln(1 - \lambda)}{\beta_0 (1 - \lambda)} + \beta_0 \ln \left(\frac{Q^2}{\mu_R^2} \right) \frac{\lambda}{1 - \lambda} \right] + \mathcal{O}(\alpha_s(\mu_R^2)^2).
\end{aligned} \tag{7.10}$$

Substituting (7.8) into the exponent of (7.9), and using (7.10), we find

$$\ln \frac{\Delta_T C_N(\alpha_s(b_0^2/b^2))}{\Delta_T C_N(\alpha_s(Q^2))} = \frac{\alpha_s(\mu_R^2)}{2\pi} \Delta_T C_N^{(1)} \frac{\lambda}{1 - \lambda} + \mathcal{O}(\alpha_s(\mu_R^2)^2). \tag{7.11}$$

Therefore, the exponent on the RHS of (7.9) corresponds to the NNLL level, according to the order counting of the contributions in the Sudakov exponent discussed below (5.22). Because the NNLL-level term was ignored in the Sudakov exponent (7.4), we can make the replacement

$$\Delta_T C_N(\alpha_s(b_0^2/b^2)) \rightarrow \Delta_T C_N(\alpha_s(Q^2)) \tag{7.12}$$

for the coefficient function in (7.6) at NLL accuracy.

The b dependence of the parton distributions in (7.6) can be reorganized similarly. In this case, the moments of the DGLAP kernel of (5.16) give the corresponding anomalous

^{*)} In the present case, actually, the moment of the coefficient function, $C_{ji,N}$, is diagonal in (j, i) and is independent of N , as seen from (7.8) and (7.3). However, even when the $C_{ji,N}$ have non-zero components for $j \neq i$ and depend on N , the results given in (7.11) and (7.12) below hold for each (j, i) -component and each moment N of $C_{ji,N}$.

dimensions $\gamma_{ij,N}(\alpha_s)$, and, for the NLO parton distributions relevant to the NLL+LO cross section, we have

$$\gamma_{ij,N}(\alpha_s) \equiv \int_0^1 dz z^{N-1} \frac{\alpha_s}{2\pi} \mathcal{P}_{ij}(z, \alpha_s) = \frac{\alpha_s}{2\pi} \gamma_{ij,N}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \gamma_{ij,N}^{(1)}, \quad (7.13)$$

where the RHS constitutes the perturbative expansion up to the two-loop term, and $\gamma_{ij,N}^{(0)}$ and $\gamma_{ij,N}^{(1)}$ are given by (A.7) and (A.8) for the transversity distributions. Then, the DGLAP equation (5.15) yields the RG evolution,

$$f_{i/h,N}(b_0^2/b^2) = \sum_{j=q,\bar{q}} \mathcal{U}_{ij,N}(b_0^2/b^2, Q^2) f_{j/h,N}(Q^2), \quad (7.14)$$

where $\mathcal{U}_{ij,N}(b_0^2/b^2, Q^2) \equiv [\mathbf{U}_N(b_0^2/b^2, Q^2)]_{ij}$ denotes the evolution operator, where

$$\mathbf{U}_N(b_0^2/b^2, Q^2) = \mathcal{P} \exp \left\{ \int_{Q^2}^{b_0^2/b^2} \frac{d\kappa^2}{\kappa^2} \gamma_N(\alpha_s(\kappa^2)) \right\}, \quad (7.15)$$

with $\gamma_{ij,N}(\alpha_s) \equiv [\gamma_N(\alpha_s)]_{ij}$, and the symbol “ \mathcal{P} ” denotes the “path ordering” expansion of the exponential matrix. Substituting (7.13) into (7.15), the result possesses the well-known structure,^{57),58)}

$$\mathbf{U}_N(b_0^2/b^2, Q^2) = [1 + \mathcal{O}(\alpha_s(b_0^2/b^2))] \exp \left\{ \frac{\gamma_N^{(0)}}{2\pi\beta_0} \ln \left(\frac{\alpha_s(Q^2)}{\alpha_s(b_0^2/b^2)} \right) \right\} [1 + \mathcal{O}(\alpha_s(Q^2))], \quad (7.16)$$

where the $\mathcal{O}(\alpha_s(b_0^2/b^2))$ term and the $\mathcal{O}(\alpha_s(Q^2))$ term involve the two-loop anomalous dimension matrix $\gamma_N^{(1)}$, and their explicit forms for the transversity distributions can be found in (A.6). Using (7.14), (7.16) and (7.10) for the parton distributions in (7.6), we find that we can replace the parton distributions in (7.6) as³⁵⁾

$$f_{i/h,N}(b_0^2/b^2) \rightarrow \sum_{j=q,\bar{q}} U_{ij,N}(b_0^2/b^2, Q^2) f_{j/h,N}(Q^2), \quad (7.17)$$

with

$$U_{ij,N}(b_0^2/b^2, Q^2) = \delta_{ij} e^{R_N(\lambda)}, \quad R_N(\lambda) \equiv \frac{\gamma_{qq,N}^{(0)}}{2\pi\beta_0} \ln(1 - \lambda), \quad (7.18)$$

up to irrelevant corrections which are down by $\alpha_s(\mu_R^2)$ and vanish for $\lambda = 0$. These corrections correspond to the NNLL-level term omitted from the Sudakov exponent (7.4). Now, (7.6) reads, to NLL accuracy,

$$\Delta_T X_{N_1, N_2}^{\text{NLL}}(Q^2, Q_T^2) = \left[1 + \frac{\alpha_s(Q^2)}{2\pi} C_F (\pi^2 - 8) \right] \delta H_{N_1, N_2}(Q^2) I_{N_1, N_2}(Q_T^2, Q^2), \quad (7.19)$$

$$I_{N_1, N_2}(Q_T^2, Q^2) \equiv \int_c^b db \frac{b}{2} J_0(bQ_T) e^{S(b, Q) + R_{N_1}(\lambda) + R_{N_2}(\lambda)}. \quad (7.20)$$

In (7.19), the large logarithms $L = \ln(Q^2 b^2 / b_0^2)$ and the associated b -integral is factorized into $I_{N_1, N_2}(Q_T^2, Q^2)$, and therefore, the all-order resummation of the large logarithmic contributions is now realized at the partonic level.

If we expand $e^{S(b, Q) + R_{N_1}(\lambda) + R_{N_2}(\lambda)}$ in powers of λ in the integrand of (7.20), similarly to (6.2) and (6.3), the most enhanced logarithmic contributions coincide with the leading tower $\alpha_s^n L^{2n}$ in (6.3), which comes from the expansion of $e^{-(A^{(1)}/2\pi)\alpha_s L^2}$ associated with the LL term, $h^{(0)}/\alpha_s$, of the Sudakov exponent (7.4), and result in the contributions $\alpha_s^n \ln^{2n-1}(Q^2/Q_T^2)/Q_T^2$ in the cross section, as shown in (6.5). Similarly, noting that [see (7.18)]

$$S(b, Q) + R_{N_1}(\lambda) + R_{N_2}(\lambda) = -\frac{A^{(1)}}{2\pi}\alpha_s L^2 - \frac{B^{(1)} + \gamma_{qq, N_1}^{(0)} + \gamma_{qq, N_2}^{(0)}}{2\pi}\alpha_s L + \mathcal{O}(\alpha_s^2), \quad (7.21)$$

the next-to-leading tower $\alpha_s^n L^{2n-1}$, which leads to $\alpha_s^n \ln^{2n-2}(Q^2/Q_T^2)/Q_T^2$, comes from, e.g., the “cross terms” $\propto (A^{(1)}\alpha_s L^2)^{n-1} \times [B^{(1)} + \gamma_{qq, N_1}^{(0)} + \gamma_{qq, N_2}^{(0)}]\alpha_s L$. For the terms corresponding to the next-to-next-to-leading tower, $\alpha_s^n L^{2n-2}$, which lead to $\alpha_s^n \ln^{2n-3}(Q^2/Q_T^2)/Q_T^2$, the coefficients $A^{(2)}$ and $\Delta_T C_N^{(1)}$ participate, in addition to $A^{(1)}$, $B^{(1)}$, $\gamma_{qq, N_1}^{(0)}$ and $\gamma_{qq, N_2}^{(0)}$. As mentioned below (6.9), to completely determine the next tower, $\alpha_s^n L^{2n-4}$, the NNLL coefficients $A^{(3)}$ and $B^{(2)}$ associated with $h^{(2)}(\lambda)$ of (5.22) and also the coefficient functions and anomalous dimensions at the two-loop level, $\Delta_T C_{ij, N}^{(2)}$ and $\gamma_{ij, N}^{(1)}$, are necessary.³⁰⁾ The NLL resummation formula given in (7.19) and (7.20) fully sums up the first three towers of logarithms contributing to the cross section, i.e., $\alpha_s^n \ln^m(Q^2/Q_T^2)$ with $m = 2n - 1, 2n - 2$ and $2n - 3$, for all n .

As discussed in §6.1, for $Q_T \gtrsim Q$, corresponding to the region (i), the resummation is irrelevant, and the cross section can be accurately obtained with fixed-order perturbation theory. We can switch between the resummed and fixed-order calculations at a certain value $Q_T \sim Q$,⁵⁹⁾ but such procedure, introducing ad-hoc boundaries between the large- Q_T and small Q_T -regions, is not convenient for the actual evaluation of the cross section. For this reason, we use (7.19) and (7.20) over the entire region of Q_T . The formula (7.20) exponentiates the functions of λ in the integrand, as a result of the large logarithmic expansion, which is valid in the regions (ii) and (iii) of §6. Thus, this exponentiated form is effective when $Q_T \lesssim Q$, corresponding to (ii) and (iii), but its behavior cannot be trusted for small $b \lesssim 1/Q$ ($Q_T \gtrsim Q$); indeed, L in (5.21) is large for small b as well as for large b , and thus (7.20) also exponentiates the unjustified large logarithmic contributions at large Q_T .^{*)} This problem can be avoided by making the replacement^{35), 55)}

$$L = \ln(Q^2 b^2 / b_0^2) \rightarrow \tilde{L} = \ln(Q^2 b^2 / b_0^2 + 1) \quad (7.22)$$

^{*)} When we expand the integrand of (7.20) in powers of α_s , the enhanced behavior of L for small b is harmless, order-by-order in α_s [see (5.29), (6.5)]. This is because the b integration in \mathcal{I}_n of (5.27) converges as $b \rightarrow 0$.

in the definition (5·21) of λ . Through this replacement, the unjustified logarithmic contributions are suppressed: as $b \rightarrow 0$, we get $L \rightarrow 0$, and $e^{S(b,Q)+R_{N_1}(\lambda)+R_{N_2}(\lambda)} \rightarrow 1$. By contrast, the resummation of the large logarithmic contributions in (7·19) and (7·20) for $b \gtrsim 1/Q$ is not affected by (7·22), since L and \tilde{L} are equivalent for organizing the large logarithmic expansion, as we have $\tilde{L} = L + \mathcal{O}(1/(Q^2 b^2))$. After this replacement, (7·19) and (7·20) read

$$\Delta_T \tilde{X}_{N_1, N_2}^{\text{NLL}}(Q^2, Q_T^2) = \left[1 + \frac{\alpha_s(Q^2)}{2\pi} C_F(\pi^2 - 8) \right] \delta H_{N_1, N_2}(Q^2) \tilde{I}_{N_1, N_2}(Q_T^2, Q^2), \quad (7.23)$$

$$\tilde{I}_{N_1, N_2}(Q_T^2, Q^2) = \int_C db \frac{b}{2} J_0(bQ_T) \left[e^{S(b,Q)+R_{N_1}(\lambda)+R_{N_2}(\lambda)} \right]_{L \rightarrow \tilde{L}}, \quad (7.24)$$

and we denote the double inverse Mellin transform of $\Delta_T \tilde{X}_{N_1, N_2}^{\text{NLL}}(Q_T^2, Q^2)$ from (N_1, N_2) space to (x_1^0, x_2^0) space as $\Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y)$, which represents the new resummed component to NLL accuracy.

The NLL resummed component obtained in this way is now combined with the fixed order cross section (4·29). Specifically, the finite component in the NLL+LO cross section is determined by (5·32), where we use (4·29) as the first term on the RHS. The result represents the NLL+LO cross section of the tDY in the $\overline{\text{MS}}$ scheme,

$$\begin{aligned} & \frac{\Delta_T d\sigma^{\text{NLL+LO}}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} \\ &= \mathcal{N} \cos(2\phi_{k_1} - \phi_1 - \phi_2) \left[\Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) + \Delta_T \tilde{Y}(Q_T^2, Q^2, y) \right], \end{aligned} \quad (7.25)$$

where

$$\Delta_T \tilde{Y}(Q_T^2, Q^2, y) \equiv \Delta_T X(Q_T^2, Q^2, y) + \Delta_T Y(Q_T^2, Q^2, y) - \Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) \Big|_{\text{fo}}. \quad (7.26)$$

Here, $(\dots)|_{\text{fo}}$ now denotes the expansion up to $\mathcal{O}(\alpha_s(\mu_R^2))$. Note that $\Delta_T \tilde{Y}(Q_T^2, Q^2, y)$ would coincide with $\Delta_T Y(Q_T^2, Q^2, y)$, if we did not perform the replacement (7·22) for $\Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y)$. We calculate the “subtraction term” $\Delta_T \tilde{X}^{\text{NLL}}|_{\text{fo}}$ in (7·26) explicitly [compare with (5·29)] and obtain

$$\begin{aligned} \Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) \Big|_{\text{fo}} &= \left[1 + \frac{\alpha_s(\mu_R^2)}{2\pi} C_F(\pi^2 - 8) \right] \tilde{\mathcal{I}}_0 \delta H(x_1^0, x_2^0; \mu_F^2) \\ &+ \frac{\alpha_s(\mu_R^2)}{2\pi} C_F \left\{ \delta H(x_1^0, x_2^0; \mu_F^2) (-\tilde{\mathcal{I}}_2 + 3\tilde{\mathcal{I}}_1) \right. \end{aligned}$$

$$\begin{aligned}
& + \left(\tilde{\mathcal{I}}_0 \ln \frac{Q^2}{\mu_F^2} - \tilde{\mathcal{I}}_1 \right) \left[\int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H \left(\frac{x_1^0}{z}, x_2^0; \mu_F^2 \right) \right. \\
& \left. + \int_{x_2^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H \left(x_1^0, \frac{x_2^0}{z}; \mu_F^2 \right) \right] \Bigg\}, \tag{7.27}
\end{aligned}$$

where

$$\tilde{\mathcal{I}}_n \equiv \int_{\mathcal{C}} db \frac{b}{2} J_0(bQ_T) \ln^n (Q^2 b^2 / b_0^2 + 1). \tag{7.28}$$

Similarly to the case of \mathcal{I}_n , (6.14), we can relate this to $\int_{\mathcal{C}} db (b/2) J_0(bQ_T) (Q^2 b^2 / b_0^2 + 1)^n$, where the contour \mathcal{C} can be deformed into that from $b = 0$ to $b = \infty$ along the real b -axis using analytic continuation for η and Cauchy's theorem. The resulting integral yields [see (C.6)]

$$\begin{aligned}
\tilde{\mathcal{I}}_0 &= \delta(Q_T^2), \\
\tilde{\mathcal{I}}_1 &= -\frac{b_0}{Q} \left[\frac{1}{Q_T} K_1 \left(\frac{b_0 Q_T}{Q} \right) \right]_{+\infty}, \tag{7.29}
\end{aligned}$$

$$\tilde{\mathcal{I}}_2 = -\frac{2b_0}{Q} \left[K_1 \left(\frac{b_0 Q_T}{Q} \right) \frac{\ln(Q/Q_T)}{Q_T} + \frac{Q}{b_0 Q_T^2} K_0 \left(\frac{b_0 Q_T}{Q} \right) \right]_{+\infty}, \tag{7.30}$$

where $K_n(x)$ denotes the modified Bessel functions of the second kind, and we have introduced the notation “ $[\cdots]_{+\infty}$ ” to indicate the “generalized + distribution” defined between $0 \leq Q_T^2 < \infty$, as $\int_0^\infty dQ_T^2 [\cdots]_{+\infty} = 0$ (see Appendix B of Ref. 35)). The quantity $\Delta_T X$ given in (4.31) and (4.32) is identical to (7.27) with $\tilde{\mathcal{I}}_n$ replaced by \mathcal{I}_n , and $\tilde{\mathcal{I}}_n$ is given by \mathcal{I}_n with the replacement $L \rightarrow \tilde{L}$ of (7.22). Because the replacement $L \rightarrow \tilde{L}$ does not affect the large b behavior of the corresponding integrand, the small Q_T behavior of $\tilde{\mathcal{I}}_n$ is the same as that of \mathcal{I}_n . In fact, we can show³⁵⁾ that for $Q_T \ll Q$, we have $\tilde{\mathcal{I}}_n = \mathcal{I}_n \times (1 + \mathcal{O}(Q_T^2/Q^2))$, i.e., $\tilde{\mathcal{I}}_n - \mathcal{I}_n = \mathcal{O}(\ln^{n-1}(Q_T^2/Q^2)/Q^2)$ [see (6.4)], and the difference $\Delta_T X - \Delta_T \tilde{X}^{\text{NLL}}|_{\text{fo}}$ is given by contributions less singular than $1/Q_T^2$ or $\delta(Q_T^2)$ as $Q_T^2 \rightarrow 0$.*) Therefore, $\Delta_T \tilde{Y}$ differs from $\Delta_T Y$ only by the $\mathcal{O}(\alpha_s)$ terms, which are less singular than $1/Q_T^2$ or $\delta(Q_T^2)$ as $Q_T^2 \rightarrow 0$; the entire effect of the replacement $L \rightarrow \tilde{L}$ is to move a portion of the “finite” component to the “singular” component in the cross section. We can rewrite (7.26) as

$$\begin{aligned}
\Delta_T \tilde{Y}(Q_T^2, Q^2, y) &= \Delta_T X^{(1)}(Q_T^2, Q^2, y) \Big|_{Q_T^2 > 0} + \Delta_T Y(Q_T^2, Q^2, y) \\
&\quad - \Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) \Big|_{\mathcal{O}(\alpha_s), Q_T^2 > 0}, \tag{7.31}
\end{aligned}$$

*) The fact that the terms proportional to $\delta(Q_T^2)$ cancel out in the difference $\Delta_T X - \Delta_T \tilde{X}^{\text{NLL}}|_{\text{fo}}$ is actually a nontrivial point. In the present NLL+LO case, they do indeed cancel, as explicitly shown in Appendix C.

where $(\dots)|_{\mathcal{O}(\alpha_s)}$ denotes the order $\alpha_s(\mu_R^2)$ term in the expansion of (\dots) . Note that $\Delta_T \tilde{Y}$ is regular over the entire region of Q_T , except for a “weak” singularity $\propto \ln(Q^2/Q_T^2)/Q^2$, which also exists in $\Delta_T Y$.^{*)} The contributions of the first and second terms in (7.31) correspond to the LO cross section (4.34), and hence, the NLL+LO cross section (7.25) is actually the NLL resummed cross section with the $\mathcal{O}(\alpha_s)$ terms of its expansion subtracted, plus the LO cross section.

One of the advantages of making the replacement $L \rightarrow \tilde{L}$ is that the NLL+LO cross section defined above satisfies the “unitarity constraint,”³⁵⁾ which means that (7.25) integrated over Q_T exactly reproduces the Q_T -integrated cross section at NLO. Noting that $e^{S(b,Q)+R_{N_1}(\lambda)+R_{N_2}(\lambda)} = 1$ for $b = 0$ [see (7.24)], we find that the Q_T -integral of $\Delta_T \tilde{X}^{\text{NLL}}$ is given by

$$\begin{aligned} \int_0^\infty dQ_T^2 \Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) &= \left[1 + \frac{\alpha_s(Q^2)}{2\pi}(\pi^2 - 8) \right] \delta H(x_1^0, x_2^0; Q^2) \\ &= \int_0^{Q^2} dQ_T^2 \Delta_T X(Q_T^2, Q^2, y), \end{aligned} \quad (7.32)$$

with $\Delta_T X(Q_T^2, Q^2, y)$ given by (4.31) and (4.32). In the second equality, we have used the relations,

$$\int_0^{Q^2} dQ_T^2 \mathcal{I}_n = \varrho_n, \quad \int_0^\infty dQ_T^2 \tilde{\mathcal{I}}_n = \delta_{n,0}, \quad (7.33)$$

with $\varrho_0 = 1$ and $\varrho_1 = \varrho_2 = 0$, which follow from (6.4) and (7.28), respectively.^{**)} For the $\Delta_T \tilde{Y}$ -term, using (7.26) and (7.33), we find

$$\begin{aligned} &\int_0^\infty dQ_T^2 \Delta_T \tilde{Y}(Q_T^2, Q^2, y) \\ &= \int_0^{Q_{T,\text{max}}^2} dQ_T^2 [\Delta_T X(Q_T^2, Q^2, y) + \Delta_T Y(Q_T^2, Q^2, y)] \\ &\quad - \int_0^\infty dQ_T^2 \Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) \Big|_{\text{fo}} \\ &= \int_{Q^2}^{Q_{T,\text{max}}^2} dQ_T^2 \Delta_T X(Q_T^2, Q^2, y) + \int_0^{Q_{T,\text{max}}^2} dQ_T^2 \Delta_T Y(Q_T^2, Q^2, y), \end{aligned} \quad (7.34)$$

^{*)} This weak singularity does not matter in $d\sigma/dQ_T$ instead of $d\sigma/dQ_T^2$.

^{**)} We note that $\varrho_3 = -4\zeta(3)$, $\varrho_4 = 0$, $\varrho_5 = -48\zeta(5)$, \dots , and the nonzero contributions in (7.33) for $n \geq 3$ come from the “contact term” proportional to $\delta(Q_T^2)$ in (6.4). Therefore, in general cases at NNLL+NLO or higher, in principle, we have to take into account those contact-term contributions to satisfy the corresponding unitarity constraint. To avoid this complication, one convenient method is to perform the matching between the resummed and the fixed-order cross sections at $Q_T > 0$ from the beginning.³⁵⁾

where $Q_{T,max}$ denotes the maximum value of Q_T determined from the partonic kinematics,

$$Q_{T,max} = Q \frac{\sqrt{[1 - (x_1^0)^2][1 - (x_2^0)^2]}}{x_1^0 + x_2^0}. \quad (7.35)$$

Adding (7.32) and (7.34), we find that the Q_T -integral of the NLL+LO cross section (7.25) is identical to that of the fixed-order Q_T differential cross section (4.29), and hence

$$\int_0^\infty dQ_T^2 \frac{\Delta_T d\sigma^{\text{NLL+LO}}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} = \frac{\Delta_T d\sigma}{dQ^2 dy d\phi_{k_1}}. \quad (7.36)$$

Here, we note that the RHS, i.e. the Q_T -integral of (4.29), is counted as the NLO cross section, because the Q_T -integrated cross section at LO is of $\mathcal{O}(\alpha_s^0)$, and its partonic subprocess is expressed by the tree diagram in Fig. 1.

7.2. The unpolarized cross section

The NLL+LO Q_T -differential cross section for the unpolarized DY process is obtained in the same way as the spin-dependent cross section in §7.1, and the results are given in Appendix A of Ref. 56) (see also Refs.^{31),32)}). The NLL+LO Q_T -differential cross section for unpolarized DY reads

$$\frac{d\sigma^{\text{NLL+LO}}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} = 2 \mathcal{N} \left[\tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) + \tilde{Y}(Q_T^2, Q^2, y) \right], \quad (7.37)$$

where the NLL resummed component \tilde{X}^{NLL} is given by

$$\begin{aligned} \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) = & \int_{\mathcal{C}} db \frac{b}{2} J_0(bQ_T) \left(e^{S(b,Q)} \left[H\left(x_1^0, x_2^0; \frac{b_0^2}{b^2}\right) \right. \right. \\ & + \frac{\alpha_s(Q^2)}{2\pi} \left\{ \int_{x_1^0}^1 \frac{dz}{z} C_{qq}^{(1)}(z) H\left(\frac{x_1^0}{z}, x_2^0; \frac{b_0^2}{b^2}\right) + \int_{x_2^0}^1 \frac{dz}{z} C_{qq}^{(1)}(z) H\left(x_1^0, \frac{x_2^0}{z}; \frac{b_0^2}{b^2}\right) \right. \\ & \left. \left. + \int_{x_1^0}^1 \frac{dz}{z} C_{qg}^{(1)}(z) K_2\left(\frac{x_1^0}{z}, x_2^0; \frac{b_0^2}{b^2}\right) + \int_{x_2^0}^1 \frac{dz}{z} C_{qg}^{(1)}(z) K_1\left(x_1^0, \frac{x_2^0}{z}; \frac{b_0^2}{b^2}\right) \right\} \right] \right)_{L \rightarrow \tilde{L}}, \end{aligned} \quad (7.38)$$

where the coefficient functions in the $\overline{\text{MS}}$ scheme are given by

$$C_{qq}^{(1)}(z) = C_F(1 - z) + C_F \left(\frac{\pi^2}{2} - 4 \right), \quad C_{qg}^{(1)}(z) = 2T_R z(1 - z), \quad (7.39)$$

with $T_R = 1/2$, and $H(x_1, x_2; \mu^2)$ and $K_{1,2}(x_1, x_2; \mu^2)$ denote the products of the NLO unpolarized parton distributions,

$$H(x_1, x_2; \mu^2) = \sum_q e_q^2 \left[q_{h_1}(x_1, \mu^2) \bar{q}_{h_2}(x_2, \mu^2) + \bar{q}_{h_1}(x_1, \mu^2) q_{h_2}(x_2, \mu^2) \right], \quad (7.40)$$

$$K_1(x_1, x_2; \mu^2) = \sum_q e_q^2 [q_{h_1}(x_1, \mu^2) + \bar{q}_{h_1}(x_1, \mu^2)] g_{h_2}(x_2, \mu^2), \quad (7.41)$$

$$K_2(x_1, x_2; \mu^2) = \sum_q e_q^2 g_{h_1}(x_1, \mu^2) [q_{h_2}(x_2, \mu^2) + \bar{q}_{h_2}(x_2, \mu^2)], \quad (7.42)$$

where $q_h(x, \mu^2)$ and $g_h(x, \mu^2)$ are the quark and gluon density distributions inside the hadron h . In (7.38), the same manipulations as for $\Delta_T \tilde{X}^{\text{NLL}}$, such as the replacement (7.22) and the contour deformation of (7.5), are applied. Also, the reorganization of the b dependence of the parton distributions according to (7.17) and (7.18) should be understood in terms of the corresponding anomalous dimensions. Note that the Sudakov exponent $S(b, Q)$ in (7.38) for the unpolarized DY process is same as that in (7.1) for the polarized one, given by (7.4). Similarly to $\Delta_T \tilde{Y}$ in (7.26), \tilde{Y} in (7.38) is defined using the finite component Y of the fixed-order α_s cross section. (The explicit expression for Y is given in Appendix A of Ref. 56.) Using logic similar to that for (7.32)-(7.36), we can show that (7.37) satisfies the unitarity condition

$$\int_0^\infty dQ_T^2 \frac{d\sigma^{\text{NLL+LO}}}{dQ^2 dQ_T^2 dy d\phi_{k_1}} = \frac{d\sigma}{dQ^2 dy d\phi_{k_1}}, \quad (7.43)$$

where the RHS denotes the NLO Q_T -integrated cross section for unpolarized DY.

§8. Asymptotic behavior of the resummed cross section at $Q_T = 0$

In §6, we discussed how the resummation formula given in (5.4) and (6.13) is controlled in each of three regions (i), (ii), and (iii) of the impact parameter b , corresponding to the kinematical regions, $Q_T \gtrsim Q$, $\Lambda_{\text{QCD}} \ll Q_T \lesssim Q$ and $Q_T \lesssim \Lambda_{\text{QCD}}$, respectively. In the region (iii), the Sudakov form factor $e^{S(b, Q)}$ with (5.22) cannot be expanded in terms of λ , since $|\lambda| = \mathcal{O}(1)$. In this section we show that the behavior of (6.13) in the region (iii), in particular its $Q_T \rightarrow 0$ limit, is controlled by the saddle point in the b integration. Such analysis has been done in Refs. 26) and 32) for the LL resummation formula. Here we extend the analysis to the case of the NLL-level resummation formula with (7.23) and (7.24). It is possible to carry out the corresponding extension on the basis of the present formalism, which realizes the resummation at the partonic level, as discussed in §7.

Let us start from the NLL resummation formula (7.24) with the exponentiation of the functions of λ in the integrand. For simplicity, we fix the renormalization scale as $\mu_R = Q$ in the following. When Q is large enough that $\alpha_s(Q^2) \ll 1$, the b integral in (7.24) at small Q_T is dominated by a saddle point, which is determined mainly by the LL term in the Sudakov exponent (7.4), $h^{(0)}(\lambda)/\alpha_s(Q^2) \rightarrow \infty$. In this case, the contributions to the b integration from very short ($|b| \ll 1/Q$) as well as very long distances ($|b| \gg 1/\Lambda_{\text{QCD}}$) along the integration contour \mathcal{C}_\pm are exponentially suppressed [see (6.13), (7.5)]. This allows us

to give up the replacement (7.22) in (7.24). Also, we may ignore the integration along the two branches, $b = b_c + e^{\pm i\theta}t$ with $t \in \{0, \infty\}$ in \mathcal{C}_{\pm} , when b_c is sufficiently large, but less than the position of the singularity in the Sudakov exponent, $b_L = (b_0/Q)e^{1/(2\beta_0\alpha_s(Q^2))}$. (In fact, the relevant integrand has a nice saddle point well below b_L (above 0) for $Q \gg \Lambda_{\text{QCD}}$, as seen from (8.6) and (8.11) below.) Then (7.24) reads, up to the exponentially suppressed corrections,

$$\tilde{I}_{N_1, N_2}(Q_T^2, Q^2) = \int_0^{b_c} db \frac{b}{2} J_0(bQ_T) \exp \left[\frac{h^{(0)}(\lambda)}{\alpha_s(Q^2)} + h^{(1)}(\lambda) + R_{N_1}(\lambda) + R_{N_2}(\lambda) \right], \quad (8.1)$$

with $b_c < b_L$. To evaluate the integral in (8.1) with the saddle-point method, we consider the simplest case, with $Q_T = 0$, and change the integration variable from b to λ of (5.21), as

$$\tilde{I}_{N_1, N_2}(Q_T^2 = 0, Q^2) = \frac{b_0^2}{4Q^2\beta_0\alpha_s(Q^2)} \int_{-\infty}^{\lambda_c} d\lambda e^{-\zeta^{(0)}(\lambda) + h^{(1)}(\lambda) + R_{N_1}(\lambda) + R_{N_2}(\lambda)}, \quad (8.2)$$

where $\lambda_c = \beta_0\alpha_s(Q^2) \ln(Q^2 b_c^2/b_0^2)$ and

$$\zeta^{(0)}(\lambda) \equiv -\frac{\lambda}{\beta_0\alpha_s(Q^2)} - \frac{h^{(0)}(\lambda)}{\alpha_s(Q^2)}. \quad (8.3)$$

Here, $\zeta^{(0)}(\lambda)$ denotes the leading term $\propto 1/\alpha_s(Q^2)$ in the exponent of (8.2), while the remaining terms, $h^{(1)}(\lambda) + R_{N_1}(\lambda) + R_{N_2}(\lambda)$, collect the contributions of the NLL level, which are down by $\alpha_s(Q^2)$ from the leading term in the relevant region, $\lambda \sim 1$. The precise position of the saddle point in the integral of (8.2) is determined by the condition

$$-\zeta^{(0)'}(\lambda) + h^{(1)'}(\lambda) + R_{N_1}'(\lambda) + R_{N_2}'(\lambda) = 0. \quad (8.4)$$

Now we express its solution as $\lambda = \lambda_{SP} + \Delta\lambda_{SP}$, where λ_{SP} is the solution of $\zeta^{(0)'}(\lambda) = 0$; i.e., λ_{SP} satisfies

$$1 - \frac{A_q^{(1)}}{2\pi\beta_0} \frac{\lambda_{SP}}{1 - \lambda_{SP}} = 0, \quad (8.5)$$

and $\Delta\lambda_{SP}$ denotes the shift of the saddle point due to the subleading terms in (8.4), $h^{(1)'}(\lambda) + R_{N_1}'(\lambda) + R_{N_2}'(\lambda)$. From (8.5), we get

$$\lambda_{SP} = \frac{2\pi\beta_0}{2\pi\beta_0 + A_q^{(1)}} \quad (8.6)$$

which is independent of Q and coincides with the saddle point at the LL level discussed in Refs. 26), 32) and 51). Substituting $A_q^{(1)} = 2C_F$ from (5.13), we obtain $\lambda_{SP} \simeq 0.6$, which is $\mathcal{O}(1)$ as it should be. Contrastingly, the shift, $\Delta\lambda_{SP} = [h^{(1)'}(\lambda_{SP}) + R_{N_1}'(\lambda_{SP}) + R_{N_2}'(\lambda_{SP})]/\zeta^{(0)''}(\lambda_{SP}) + \dots$, is $\mathcal{O}(\alpha_s(Q^2))$ or higher. Up to the exponentially suppressed

corrections, the saddle point evaluation of (8.2) around $\lambda = \lambda_{SP} + \Delta\lambda_{SP} \equiv \omega$ can be performed in the usual way. Defining $\xi(\lambda) \equiv \zeta^{(0)}(\lambda) - h^{(1)}(\lambda) - R_{N_1}(\lambda) - R_{N_2}(\lambda)$, we obtain

$$\begin{aligned}
\tilde{I}_{N_1, N_2}(0, Q^2) &= \frac{b_0^2}{4Q^2\beta_0\alpha_s(Q^2)} e^{-\xi(\omega)} \int_{-\infty}^{\infty} d\lambda e^{-[\xi''(\omega)/2](\lambda-\omega)^2} \\
&\quad \times \left(1 - \frac{1}{24}\xi'''(\omega)(\lambda-\omega)^4 + \frac{1}{72}[\xi'''(\omega)]^2(\lambda-\omega)^6 + \dots \right) \\
&= \frac{b_0^2}{4Q^2\beta_0\alpha_s(Q^2)} e^{-\xi(\omega)} \sqrt{\frac{2\pi}{\xi''(\omega)}} \left(1 - \frac{1}{8} \frac{\xi'''(\omega)}{[\xi''(\omega)]^2} + \frac{5}{24} \frac{[\xi'''(\omega)]^2}{[\xi''(\omega)]^3} + \dots \right).
\end{aligned} \tag{8.7}$$

Because $\xi(\omega) \sim 1/\alpha_s(Q^2)$ for $\alpha_s(Q^2) \ll 1$, the second and third terms in the parentheses on the RHS of (8.7) are of $\mathcal{O}(\alpha_s(Q^2))$, and similarly it is straightforward to show that the subsequent terms denoted by the ellipses are of $\mathcal{O}(\alpha_s(Q^2)^2)$ or higher. We ignore the second and following terms in the parentheses, because they correspond to the NNLL or higher-level contributions that were omitted from our starting point (7.24). As discussed below (5.22), the NNLL contributions are associated with the terms $\alpha_s^{n+1}L^n \propto \alpha_s\lambda^n$ ($n \geq 1$) in the exponent of the resummation formula. More precisely, in the regions (i) and (ii) of §6, those NNLL contributions behave as $\sim \alpha_s^2L, \alpha_s^3L^2, \dots$, while at the saddle point $\lambda = \omega = \lambda_{SP} + \Delta\lambda_{SP} \sim 1$, they would behave as $\sim \alpha_s$. A similar situation occurs for the NNNLL and higher-level contributions. As mentioned in §6.3, this indicates that, for $Q_T \approx 0$ in the region (iii), all contributions are completely controlled by the single small parameter $\alpha_s(Q^2)$, and we have to use a classification with respect to the order counting of the terms that is different from the customary and all-order resummed perturbation theory used in the regions (i) and (ii). In particular, when we start with the NLL-level resummation formula and go to the $Q_T \approx 0$ region, we should ignore the contributions that are of the same order as the $\mathcal{O}(\alpha_s)$ terms in the exponent of the corresponding formula.^{32), 56)}

Substituting $\omega = \lambda_{SP} + \Delta\lambda_{SP}$ and $\xi = \zeta^{(0)} - h^{(1)} - R_{N_1} - R_{N_2}$ into (8.7), and expanding the result in terms of $\Delta\lambda_{SP}$, we find

$$\tilde{I}_{N_1, N_2}(0, Q^2) = \frac{b_0^2}{4Q^2\beta_0\alpha_s(Q^2)} \sqrt{\frac{2\pi}{\zeta^{(0)''}(\lambda_{SP})}} e^{-[\zeta^{(0)}(\lambda_{SP}) + h^{(1)}(\lambda_{SP}) + R_{N_1}(\lambda_{SP}) + R_{N_2}(\lambda_{SP})]}, \tag{8.8}$$

to NLL accuracy. Note that using $\zeta^{(0)'}(\lambda_{SP}) = 0$, the shift $\Delta\lambda_{SP}$ yields only NNLL-level corrections. We substitute this into (7.23) and combine the result with the coefficient function and the parton distributions. Then, using the same logic as above, the $\mathcal{O}(\alpha_s(Q^2))$ term in the coefficient function in (7.23) is also ignored as $[1 + \alpha_s(Q^2)C_F(\pi^2 - 8)/2\pi] \rightarrow 1$, and

thus we have

$$\Delta_T \tilde{X}_{N_1, N_2}^{\text{NLL}}(0, Q^2) = \delta H_{N_1, N_2}(Q^2) \tilde{I}_{N_1, N_2}(0, Q^2) . \quad (8.9)$$

Then, performing the double inverse Mellin transformation to the (x_1^0, x_2^0) space, we obtain⁵⁶⁾

$$\begin{aligned} \Delta_T \tilde{X}^{\text{NLL}}(0, Q^2, y) \\ = \left(\frac{b_0^2}{4Q^2 \beta_0 \alpha_s(Q^2)} \sqrt{\frac{2\pi}{\zeta^{(0)''}(\lambda_{SP})}} e^{-\zeta^{(0)}(\lambda_{SP}) + h^{(1)}(\lambda_{SP})} \right) \delta H \left(x_1^0, x_2^0; \frac{b_0^2}{b_{SP}^2} \right), \end{aligned} \quad (8.10)$$

with

$$b_{SP} = \frac{b_0}{Q} e^{\lambda_{SP}/[2\beta_0 \alpha_s(Q^2)]}, \quad (8.11)$$

which is the value of b corresponding to λ_{SP} in (8.6). Here, we have used the fact that $e^{R_{N_1}(\lambda_{SP})}$ and $e^{R_{N_2}(\lambda_{SP})}$ in (8.8) can be identified with the NLO evolution operators from the scale Q to b_0/b_{SP} , to the present accuracy. This yields $\delta H(x_1^0, x_2^0; Q^2)$ with the scale shifted as $Q \rightarrow b_0/b_{SP}$ [see (7.17), (7.18)].

The saddle-point evaluation of the NLL resummation formula (7.38) for the unpolarized case can be performed similarly, and the result is given by⁵⁶⁾

$$\begin{aligned} \tilde{X}^{\text{NLL}}(0, Q^2, y) \\ = \left(\frac{b_0^2}{4Q^2 \beta_0 \alpha_s(Q^2)} \sqrt{\frac{2\pi}{\zeta^{(0)''}(\lambda_{SP})}} e^{-\zeta^{(0)}(\lambda_{SP}) + h^{(1)}(\lambda_{SP})} \right) H \left(x_1^0, x_2^0; \frac{b_0^2}{b_{SP}^2} \right), \end{aligned} \quad (8.12)$$

up to NLL accuracy, which corresponds to the above result (8.10) with the replacement $\delta H(x_1^0, x_2^0; b_0^2/b_{SP}^2) \rightarrow H(x_1^0, x_2^0; b_0^2/b_{SP}^2)$. At the present accuracy, the gluon distribution decouples for $Q_T \approx 0$, since the corresponding coefficient function $C_{qg}^{(1)}$ in (7.39) is of order $\alpha_s(Q^2)$.

We emphasize that the results (8.10)-(8.12) are exact, up to the NNLL corrections corresponding to the $\mathcal{O}(\alpha_s(Q^2))$ effects. We also see that these results give the explicit realization of the “degree 0” approximation discussed in §6.3 for polarized and unpolarized DY processes. The common factor for both the polarized and unpolarized results, given by the contribution in the parentheses of (8.10) and (8.12), represents “very large perturbative effects” due to the universal Sudakov factor. Substituting (8.6) and the running coupling constant at two-loop level [see (5.19)] into this common factor, we obtain, to the present accuracy,

$$\frac{b_0^2}{4Q^2 \beta_0 \alpha_s(Q^2)} \sqrt{\frac{2\pi}{\zeta^{(0)''}(\lambda_{SP})}} e^{-\zeta^{(0)}(\lambda_{SP}) + h^{(1)}(\lambda_{SP})}$$

$$\begin{aligned}
&= \frac{b_0^2 \sqrt{\pi a}}{2\sqrt{2}(a+1)} \frac{\sqrt{\ln(Q^2/\Lambda_{\text{QCD}}^2)}}{\Lambda_{\text{QCD}}^2} \left(\frac{\Lambda_{\text{QCD}}^2}{Q^2} \right)^{a \ln(1+1/a)} \\
&\quad \times [\ln(Q^2/\Lambda_{\text{QCD}}^2)]^{(\beta_1/\beta_0^2)[1-a \ln(1+1/a)]} e^{h^{(1)}(\lambda_{SP})}, \tag{8.13}
\end{aligned}$$

with $a \equiv A_q^{(1)}/(2\pi\beta_0) \simeq 0.6$. Here, the factors in the second line reproduce the well-known asymptotic behavior for $Q \gg \Lambda_{\text{QCD}}$, derived from the conventional LL-level saddle-point calculation²⁶⁾ (see also Refs. 51) and 52)), while those in the third line represent the NLL-level effects. We note that the subleading, NLL contributions do not disappear in the asymptotic limit $Q \rightarrow \infty$; e.g., $e^{h^{(1)}(\lambda_{SP})}$ provides the $\mathcal{O}(1)$ constant factor [see (5.24)]. In fact, according to the discussion of the “degree N ” approximation in §6.3, our results (8.10)-(8.13), representing the degree 0 approximation, are exact in the asymptotic limit, while the “degree -1 ” approximation, which corresponds to the LL resummation, is not.³²⁾

In our saddle-point formulae (8.10) and (8.12), the “new scale” b_0/b_{SP} participating in the parton distributions is another NLL-level effect, which does not disappear in the asymptotic limit $Q \rightarrow \infty$. In fact, for the polarized case (8.10), the corresponding contribution is associated with the $\mathcal{O}(1)$ terms, $R_{N_1}(\lambda_{SP}) + R_{N_2}(\lambda_{SP})$, in the exponent of (8.8). Substituting (8.6), we can express (8.11) as

$$\frac{b_0}{b_{SP}} = \Lambda_{\text{QCD}} \left(\frac{Q}{\Lambda_{\text{QCD}}} \right)^{\frac{a}{a+1}} [\ln(Q^2/\Lambda_{\text{QCD}}^2)]^{-\frac{\beta_1}{2\beta_0^2(a+1)}}, \tag{8.14}$$

where the first two factors on the RHS coincide with the corresponding LL-level expression^{32), 51)} of b_0/b_{SP} , while the third one, involving the logarithms of $Q^2/\Lambda_{\text{QCD}}^2$, represents the NLL-level modification induced by the two-loop running of the coupling constant in (8.11). Because $a \simeq 0.6$, this implies $b_0/b_{SP} \sim \Lambda_{\text{QCD}} (Q/\Lambda_{\text{QCD}})^{a/(a+1)} \simeq \sqrt{\Lambda_{\text{QCD}} Q}$, modulo logarithms. Therefore, our saddle-point formulae (8.10) and (8.12) are directly applicable to the production of very high mass DY pairs such that $b_0/b_{SP} \gg \Lambda_{\text{QCD}}$. In fact, in such cases, the integral over b in our resummation formula (7.24) is actually dominated by the region of b near b_{SP} ($\ll 1/\Lambda_{\text{QCD}}$), where the form of the integrand in (7.24) is accurate [see the discussion below (5.10)]. Practically, however, the polarized DY experiments at RHIC, J-PARC, GSI, etc., will measure dileptons with masses $Q \sim$ several GeV, where b_0/b_{SP} may be of order 1 GeV, and our formulae (8.10) and (8.12) are not useful for analyzing those cases. Apparently, a similar problem arises at the LL-level,^{26), 32), 52)} and the present result indicates that the higher-order perturbative corrections at NLL level cannot solve this problem.

The root of this frustrating conclusion is in the fact that the integrand of our resummation formula (7.24) is composed of purely perturbative quantities and is inaccurate in the long-distance region (iii), $|b| \gtrsim \Lambda_{\text{QCD}}$. We have to introduce nonperturbative effects relevant to

such a large b region in order to treat the small Q_T behavior in the tDY for the kinematics of the relevant experiments. We discuss this point and the application of the results to phenomenology in the next section.

§9. Predictions for the transversely polarized Drell-Yan process at RHIC and J-PARC in the NLL+LO Q_T -resummation framework

On the basis of the QCD factorization and the transverse-momentum resummation framework, we have obtained all the necessary analytic formulae of the higher-order perturbative corrections for the QCD prediction of the dilepton Q_T spectrum and the spin asymmetry in the transversely polarized DY process. In this section, we specify the nonperturbative inputs necessary for estimating them and calculate those observables numerically with the kinematics of an ongoing experiment at RHIC and a possible future experiment at J-PARC. A detailed numerical study has already been given in Ref. 56), and here we highlight the important results to show how the soft-gluon resummation contributions control the behavior of the Q_T spectrum and the double transverse-spin asymmetry, especially in the small Q_T region. We also analyze the double-spin asymmetries, utilizing the saddle-point evaluation of the resummation formula.

9.1. The Q_T -differential cross sections

First, we calculate the Q_T -differential cross sections (7.25) and (7.37) for polarized and unpolarized Drell-Yan processes, respectively, with the kinematics of pp collisions at RHIC. For simplicity, here and below we denote the parton distributions of the proton as [see (2.4), (A.1), (7.40)-(7.42)]

$$\delta q(x, \mu^2) \equiv \delta q_{h=p}(x, \mu^2), \quad q(x, \mu^2) \equiv q_{h=p}(x, \mu^2), \quad g(x, \mu^2) \equiv g_{h=p}(x, \mu^2), \quad (9.1)$$

omitting the subscript, and also we denote the combination of the azimuthal angles appearing in (7.25) as $2\phi_{k_1} - \phi_1 - \phi_2 \equiv 2\phi$.

To calculate the unpolarized cross section (7.37), we need nonperturbative inputs for the unpolarized quark and gluon distributions, $q(x, \mu^2)$ and $g(x, \mu^2)$; we use the NLO GRV98 distributions for them.⁶⁴⁾ To calculate the polarized cross section (7.25), we need nonperturbative inputs for the transversity $\delta q(x, \mu^2)$. Recently, Anselmino et al.⁶⁰⁾ gave the first estimation of the transversity distributions from semi-inclusive DIS (SIDIS) data, combined with e^+e^- data for the associated (Collins) fragmentation function. They performed a global fit of the SIDIS spin-asymmetry data with the corresponding LO QCD formula, assuming that the antiquark distributions vanish, i.e. $\delta \bar{q}(x, \mu^2) = 0$, and obtained the LO quark transversity distribution. Thus, their results cannot be used for the present calculation of

tDY at the NLL+LO accuracy, in which the *NLO* transversity distributions for *both the quark and antiquark* are necessary. At the moment, no other experimental information on the transversity exists^{*)} Theoretically, a nontrivial relation among the transversity $\delta q(x)$, unpolarized $q(x)$, and longitudinally polarized $\Delta q(x)$ distributions is provided by the Soffer inequality,⁶³⁾

$$2|\delta q(x)| \leq q(x) + \Delta q(x), \quad (9.2)$$

for each flavor of quark and antiquark. In this paper, we use a model of the NLO transversity distributions which satisfy the Soffer inequality, following Refs.20) and 22). In this model, we saturate the Soffer bound of (9.2) at the low input scale $\mu_0 \simeq 0.6$ GeV, as $\delta q(x, \mu_0^2) = [q(x, \mu_0^2) + \Delta q(x, \mu_0^2)]/2$. Here, we cannot determine the signs of the distributions from (9.2), and in this calculation, we have chosen all the signs to be positive. For the input functions on the RHS of this formula, $q(x, \mu_0^2)$ and $\Delta q(x, \mu_0^2)$, we use the NLO GRV98⁶⁴⁾ and GRSV2000 (“standard scenario”)⁶⁵⁾ distributions, respectively. Then we evolve $\delta q(x, \mu_0^2)$ to higher scales, according to the NLO DGLAP kernel for the transversity as (A.6).

We also introduce another nonperturbative contribution to the Q_T -differential cross sections. As mentioned in §8, in the kinematical regions for most experiments, including those at RHIC and J-PARC, the small Q_T behavior of our resummation formula (7.24) is dominated by the contribution from $|b| \gtrsim 1/\Lambda_{\text{QCD}}$ in the b integral, which corresponds to the region (iii) of §6.3. However, the integrand of (7.24) involving purely perturbative quantities is not accurate for such a large $|b|$ region, and the corresponding long-distance behavior has to be complemented by relevant nonperturbative effects. Formally, those nonperturbative effects play a role of compensating for the ambiguity that the prescription for the b integration in (7.24) needed to avoid the singularity in the Sudakov exponent $S(b, Q)$ of (7.4) is not unique, such as the b_* prescription of (6.10), (6.11), and the present prescription with (6.13). Therefore, following Refs. 32), 45) and 35), in (7.24) we make the replacement

$$e^{S(b, Q)} \rightarrow e^{S(b, Q) - g_{NP} b^2}, \quad (9.3)$$

with a nonperturbative parameter g_{NP} . Because exactly the same Sudakov factor, $e^{S(b, Q)}$, participates in the corresponding formula for the unpolarized case, (7.38), as noted in §7.2, we perform the replacement (9.3) with the same nonperturbative parameter g_{NP} in the NLL+LO unpolarized differential cross section. This may be interpreted as assuming the same “intrinsic transverse momentum” of partons inside nucleons for both polarized and

^{*)} For a recent result of a lattice QCD simulation of the first and second moments of the u - and d -quark transversity distributions, see Ref. 61). The transversity distributions fitted to the prediction of the chiral quark soliton model of nucleons are given in Ref. 62).

unpolarized cases, corresponding to the Gaussian smearing factor of (9.3). For the present calculations, we use $g_{NP} = 0.5 \text{ GeV}^2$, which is consistent with a study of the Q_T spectrum in the unpolarized case.³⁶⁾

Assuming $e^{-g_{NP}b^2} \rightarrow 1$ in customary perturbation theory, it is straightforward to show that all results derived in §7 are unchanged under the replacement (9.3). In particular, the NLL+LO Q_T -differential cross sections (7.25) and (7.37) with (9.3) satisfy the unitarity condition, given in (7.36) and (7.43), exactly.

For all the following numerical calculations, we choose $\phi = 0$ for the azimuthal angle of one of the leptons, $\mu_F = \mu_R = Q$ for the factorization and renormalization scales, and $b_c = 0$ and $\theta = \frac{7}{32}\pi$ for the integration contour \mathcal{C}_\pm defined below (6.13).

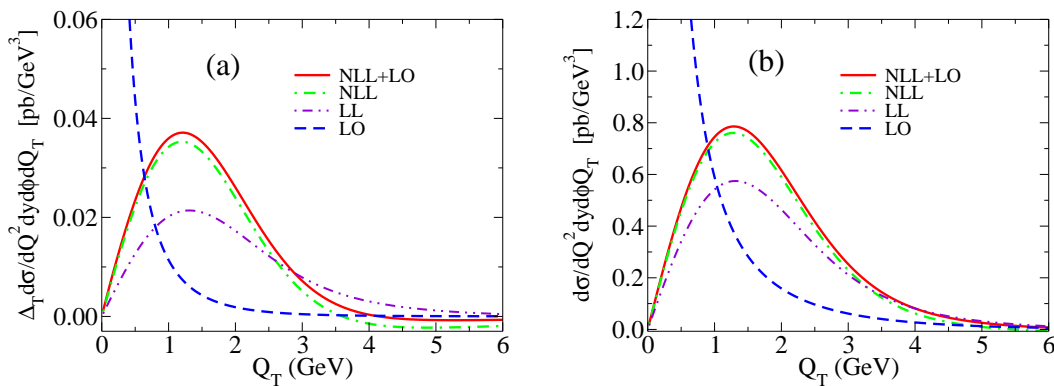


Fig. 2. The spin-dependent and spin-averaged differential cross sections for tDY: (a) $\Delta_T d\sigma/dQ^2 dQ_T dy d\phi$ and (b) $d\sigma/dQ^2 dQ_T dy d\phi$, as functions of Q_T with RHIC kinematics, $\sqrt{S} = 200 \text{ GeV}$, $Q = 5 \text{ GeV}$, $y = 2$ and $\phi = 0$, with $g_{NP} = 0.5 \text{ GeV}^2$.

The tDY Q_T -differential cross sections for the spin-dependent and spin-averaged cases are shown in Fig. 2(a) and Fig. 2(b), respectively, for $\sqrt{S} = 200 \text{ GeV}$, $Q = 5 \text{ GeV}$ and $y = 2$, corresponding to the detection of dileptons with the PHENIX detector at RHIC. The solid curve in Fig. 2(a) shows the NLL+LO differential cross section (7.25), multiplied by $2Q_T$. The dot-dashed curve represents the contribution from the NLL resummed component $\Delta_T \tilde{X}^{\text{NLL}}$ in (7.25).*) The LO cross section (4.34) is depicted by the dashed curve, which becomes large and diverges as $Q_T \rightarrow 0$, due to the singular terms $\alpha_s \ln(Q^2/Q_T^2)/Q_T^2$.

*) In principle, we should take the “degree 0” approximation for the numerical calculations when $Q_T \approx 0$, as discussed in §6 and §8. But we do not make the corresponding replacement for the coefficient functions, $C_{ji}(z, \alpha_s) \rightarrow \delta_{ji}\delta(1-z)$, for $Q_T \approx 0$ in our numerical calculations, because the effect of the replacement is invisible in the solid as well as dot-dashed curve for $Q_T \approx 0$ in Fig. 2(a), and also in Fig. 2(b) below. For the spin asymmetry, which is defined as the ratio of the cross sections in Figs. 2(a) and (b) and is discussed in §9.2, the replacement would increase the NLL+LO asymmetry (9.4) and the NLL asymmetry (9.5) for $Q_T \approx 0$ in Figs.3-5 by about 5%.

and α_s/Q_T^2 in $\Delta_T X^{(1)}$ of (4.32). This singular behavior disappears in the NLL resummed component $\Delta_T \tilde{X}^{\text{NLL}}$ when we sum up the recoil logarithms to all orders in α_s . The NLL resummed component dominates the NLL+LO cross section in the peak region of the solid curve, i.e., at intermediate as well as small values of Q_T . As a result, the NLL+LO cross section (7.25) is finite and well behaved over all regions of Q_T . We also show the LL result, represented by the two-dot-dashed curve, obtained from the corresponding NLL result by omitting the contributions from the NLL terms, i.e., $h^{(1)}(\lambda)$, $R_{N_1}(\lambda)$, $R_{N_2}(\lambda)$ in (7.24) and $\alpha_s(Q^2)C_F(\pi^2 - 8)/2\pi$ in (7.23). The LL contributions are sufficient to obtain a finite cross section, suppressing the cross sections for $Q_T \approx 0$.

The curves in Fig. 2(b) are the same as those in Fig. 2(a), but for the unpolarized cross sections; e.g., the solid curve is obtained by calculating (7.37), and the dot-dashed curve is the contribution from the NLL resummed component \tilde{X}^{NLL} in (7.37). The behavior of each curve is similar to that in the polarized case.

Comparing the NLL results with the corresponding LL results in Figs. 2(a) and (b), we find that the contributions from the NLL terms considerably enhance the cross sections in both polarized and unpolarized cases, while the enhancement is larger for the former case: The “universal” term $h^{(1)}(\lambda)$ gives a similar (enhancement) effect for both channels, while the other NLL contributions associated with the evolution operators and the $\mathcal{O}(\alpha_s)$ terms in the coefficient functions give different effects in the polarized and unpolarized cases [see (7.18), (7.3) and (7.39)].

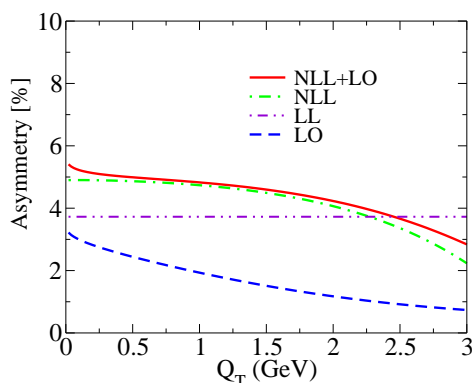


Fig. 3. The asymmetries for RHIC kinematics, $\sqrt{S} = 200$ GeV, $Q = 5$ GeV, $y = 2$ and $\phi = 0$, obtained from each curve in Fig. 2.

9.2. The double transverse-spin asymmetries

We consider the double transverse-spin asymmetry, defined by the ratio of the NLL+LO cross sections (7.25) and (7.37),

$$\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T) = \frac{1}{2} \cos(2\phi) \frac{\Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) + \Delta_T \tilde{Y}(Q_T^2, Q^2, y)}{\tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) + \tilde{Y}(Q_T^2, Q^2, y)}, \quad (9.4)$$

as a function of Q_T . Figure 3 plots the asymmetries for the tDY at RHIC, given by the ratio of the value of each line in Fig. 2(a) to that of the corresponding line in Fig. 2(b); that is, the solid curve gives the NLL+LO asymmetry $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ of (9.4), and the dot-dashed curve gives the NLL result, defined as

$$\mathcal{A}_{TT}^{\text{NLL}}(Q_T) = \frac{1}{2} \cos(2\phi) \frac{\Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y)}{\tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y)}. \quad (9.5)$$

The NLL+LO asymmetry $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ in Fig. 3 is almost flat in the peak region of the NLL+LO cross sections in Fig. 2. This flat behavior is dominated by the NLL resummed components, and it reflects the fact that the soft gluon effects resummed into the Sudakov factor $e^{S(b,Q)}$ with (7.4) are universal to the NLL accuracy between the numerator and the denominator of $\mathcal{A}_{TT}^{\text{NLL}}(Q_T)$ of (9.5).*) A slight increase of $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ as $Q_T \rightarrow 0$ is due to the terms $\propto \ln(Q^2/Q_T^2)$ in the “regular components” $\Delta_T \tilde{Y}$ and \tilde{Y} in (9.4). The dashed curve in Fig. 3 shows the LO asymmetry $\mathcal{A}_{TT}^{\text{LO}}(Q_T)$, defined by the ratio of (4.34) to the corresponding LO cross section for unpolarized DY process,

$$\mathcal{A}_{TT}^{\text{LO}}(Q_T) = \frac{1}{2} \cos(2\phi) \frac{\Delta_T X^{(1)}(Q_T^2, Q^2, y)|_{Q_T^2 > 0} + \Delta_T Y(Q_T^2, Q^2, y)}{X^{(1)}(Q_T^2, Q^2, y)|_{Q_T^2 > 0} + Y(Q_T^2, Q^2, y)}, \quad (9.6)$$

where $X^{(1)}$ denotes the singular part of the $\mathcal{O}(\alpha_s)$ unpolarized cross section,³¹⁾ defined similarly to $\Delta_T X^{(1)}$ in (4.32). Although both the numerator and denominator of (9.6) diverge as $Q_T \rightarrow 0$, as shown in Figs. 2(a) and (b), $\mathcal{A}_{TT}^{\text{LO}}(Q_T)$, their ratio, appears to be finite. However, this LO result decreases as Q_T increases, and it is much smaller than the NLL+LO result, indicating that the soft gluon resummation is crucial for the prediction of the asymmetries at small Q_T and intermediate Q_T . The two-dot-dashed curve in Fig. 3 shows the LL result [see (7.23), (7.24), (7.38)],

$$\mathcal{A}_{TT}^{\text{LL}}(Q_T) = \frac{1}{2} \cos(\phi) \frac{\delta H(x_1^0, x_2^0; Q^2)}{H(x_1^0, x_2^0; Q^2)}, \quad (9.7)$$

*) We note that the shape of the Q_T spectra in Figs. 2(a) and (b) is actually sensitive to the value of the nonperturbative parameter g_{NP} of (9.3) (see Ref. 23)). But the corresponding g_{NP} dependence almost cancels between the numerator and denominator in the asymmetries of (9.4) and (9.5) in the range $g_{NP} = 0.3\text{--}0.8 \text{ GeV}^2$.⁵⁶⁾

which is constant in Q_T , because the soft-gluon corrections in the numerator and denominator exactly cancel at the LL level [see also (6.6)]. As expected from Fig. 2, $\mathcal{A}_{TT}^{\text{NLL}}(Q_T)$ is larger than $\mathcal{A}_{TT}^{\text{LL}}(Q_T)$, due to the effects of the NLL contributions. In particular, among the NLL-level effects noted at the end of §9.1, the contributions associated with the evolution operators play an important role to amplify $\mathcal{A}_{TT}^{\text{NLL}}(Q_T)$ in comparison with $\mathcal{A}_{TT}^{\text{LL}}(Q_T)$: In the resummation formula (7.24), the large logarithmic contributions due to the Sudakov factor in (7.4) and (5.8) strongly enhance the contributions from the region $b \sim 1/Q_T$ in the b integration, where the evolution operator (7.18) causes the scale of the parton distributions to be $b_0/b \sim Q_T$, instead of Q . Therefore, the numerator of (9.5) is dominated by the transversity distributions at the scale $\sim Q_T$. A similar mechanism arises in the denominator, given by (7.38), so that, after the cancellation of the universal Sudakov factor between the numerator and denominator, the dominant contribution to (9.5) is given by $\mathcal{A}_{TT}^{\text{NLL}}(Q_T) \sim [\cos(2\phi)/2] \delta H(x_1^0, x_2^0; Q_T^2)/H(x_1^0, x_2^0; Q_T^2)$. Note that the transversity distributions in (2.4) obey QCD evolution of the non-singlet type, described by (A.6). Thus, as the scale μ increases, the antiquark transversity distribution $\delta\bar{q}(x, \mu^2)$ in the small x region grows less rapidly than the unpolarized sea distribution $\bar{q}(x, \mu^2)$ in (7.40). The latter grows rapidly for small x through mixing with the gluon distribution. Therefore, the ratio of the sea distributions $\delta\bar{q}(x, \mu^2)/\bar{q}(x, \mu^2)$ at small x becomes enhanced as the scale μ is reduced from Q to Q_T . This explains why the NLL asymmetry $\mathcal{A}_{TT}^{\text{NLL}}(Q_T) \sim [\cos(2\phi)/2] \delta H(x_1^0, x_2^0; Q_T^2)/H(x_1^0, x_2^0; Q_T^2)$ is enhanced in comparison with the LL result $\mathcal{A}_{TT}^{\text{LL}}(Q_T)$ appearing in (9.7) in the present case, with $x_1^0 = 0.185$ and $x_2^0 = 0.003$ in (4.7) [see also the discussion in §9.3 using the saddle-point formula (9.13) below].

Now we compare these asymmetries in Fig. 3 with the asymmetry for the Q_T -integrated cross sections A_{TT} , defined as

$$\begin{aligned} A_{TT} &\equiv \frac{\int dQ_T^2 [\Delta_T d\sigma^{\text{NLL+LO}}/dQ^2 dQ_T^2 dy d\phi]}{\int dQ_T^2 [d\sigma^{\text{NLL+LO}}/dQ^2 dQ_T^2 dy d\phi]} \\ &= \frac{[\Delta_T d\sigma/dQ^2 dy d\phi]}{[d\sigma/dQ^2 dy d\phi]} = \frac{1}{2} \cos(2\phi) \frac{\delta H(x_1^0, x_2^0; Q^2) + \dots}{H(x_1^0, x_2^0; Q^2) + \dots}, \end{aligned} \quad (9.8)$$

where we have used the fact that our NLL+LO Q_T -differential cross sections satisfy the unitarity conditions (7.36) and (7.43) exactly, and the ellipses on the RHS stand for the NLO $[\mathcal{O}(\alpha_s)]$ correction terms in the $\overline{\text{MS}}$ scheme. Therefore, A_{TT} defined in this way coincides with the NLO asymmetry calculated in Ref. 20).^{*)} Using the present nonperturbative inputs, we get $A_{TT} = 4.0\%$. This is close to the value of $\mathcal{A}_{TT}^{\text{LL}}(Q_T)$ and is smaller than that of

^{*)} In Ref. 20), the corresponding asymmetry is defined through a certain integration over ϕ , and it is equal to (9.8) with the formal replacement $\cos(2\phi) \rightarrow 2/\pi$. Also, the input parton distributions in Ref. 20) are not completely the same as ours (see the discussion in Ref. 56)).

$\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ in the flat region, which is reasonable, because A_{TT} in (9.8) coincides with $\mathcal{A}_{TT}^{\text{LL}}(Q_T)$ in (9.7), up to NLO QCD corrections.

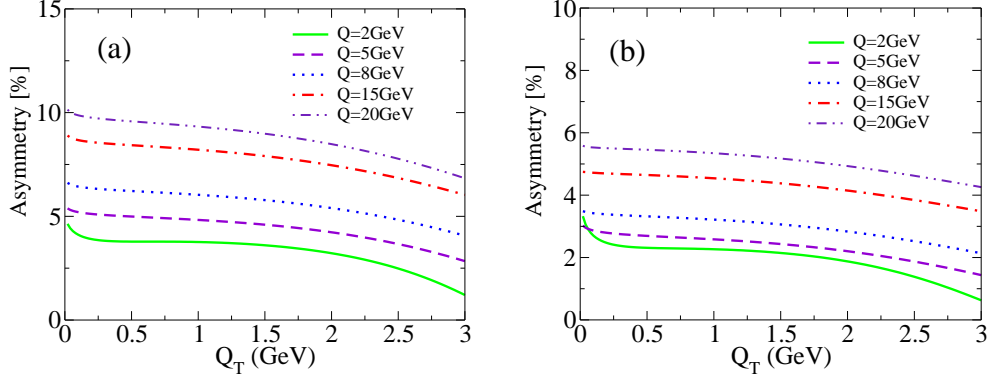


Fig. 4. The NLL+LO $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ in (9.4) with (9.3) using $g_{NP} = 0.5\text{GeV}^2$ with RHIC kinematics: (a) $\sqrt{S} = 200$ GeV, $y = 2$ and $\phi = 0$; (b) $\sqrt{S} = 500$ GeV, $y = 2$ and $\phi = 0$.

Figures 4(a) and (b) show the NLL+LO asymmetries $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ with RHIC kinematics, $\sqrt{S} = 200$ GeV and $\sqrt{S} = 500$ GeV, respectively, with various values of Q and $y = 2$. The dashed curve in Fig. 4(a) is the same as the solid curve in Fig. 3, and similarly to this dashed curve, all other curves in Figs. 4(a) and (b) exhibit the characteristic flat behavior in the small Q_T region. We mention that the dependence of $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ on Q comes from the small- x behavior of the relevant parton distributions: A smaller Q corresponds to a smaller $x_{1,2}^0 = e^{\pm y}Q/\sqrt{S}$, so that the small- x rise of the unpolarized sea distributions enhances the denominator of (9.4). A similar mechanism also explains why the results for $\sqrt{S} = 500$ GeV are smaller than those for $\sqrt{S} = 200$ GeV. Comparing with the NLO A_{TT} in (9.8), we find⁵⁶⁾ that the values of $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ in the flat region of Figs. 4(a) and (b) are larger by 20-30% than the corresponding values of A_{TT} .

Next, we present the results for J-PARC kinematics, i.e., the results for the tDY in pp collisions at the moderate CM energy $\sqrt{S} = 10$ GeV. Figure 5(a) shows the double transverse-spin asymmetries for $\sqrt{S} = 10$ GeV, $Q = 2$ GeV and $y = 0$, and the curves have the same meanings as the corresponding curves in Fig. 3. The results for J-PARC have features similar to those in Fig. 3 for RHIC: The NLL+LO asymmetry $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ is flat in the small Q_T region, and it is dominated by the NLL asymmetry $\mathcal{A}_{TT}^{\text{NLL}}(Q_T)$; the NLL asymmetry is enhanced in comparison with the LL result, and the fixed-order LO result is far below the others. We note that $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T) \simeq 15\%$ in the flat region is again larger than the corresponding prediction using (9.8), $A_{TT} = 12.8\%$. Figure 5(b) shows the NLL+LO asymmetries $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ with J-PARC kinematics, $\sqrt{S} = 10$ GeV, $Q = 2, 2.5, 3.5$ GeV, and $y = 0$, and the solid curve is the same as the solid curve in Fig. 5(a). We observe that

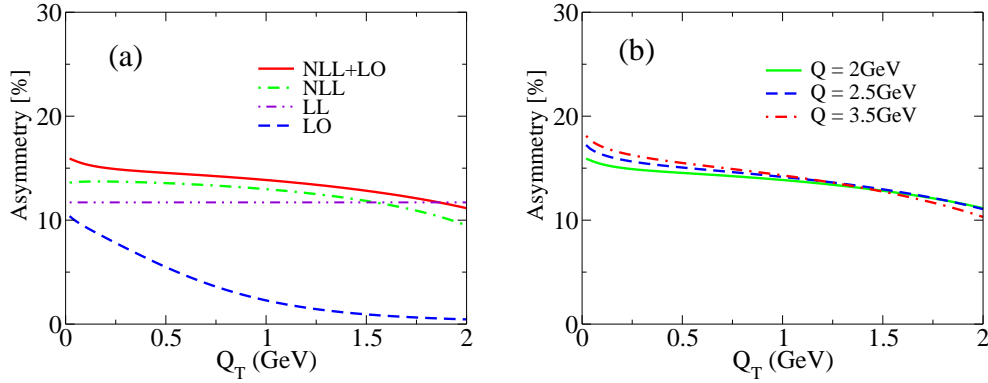


Fig. 5. The asymmetries with J-PARC kinematics. (a) The asymmetries obtained similarly to Fig. 3(a), with $\sqrt{S} = 10$ GeV, $Q = 2$ GeV, $y = 0$ and $\phi = 0$. (b) The NLL+LO $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ with $\sqrt{S} = 10$ GeV, $y = 0$ and $\phi = 0$.

the asymmetries $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ are flat and are approximately 15% for all Q ; we note that this value of $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ is larger by 20-30% than the corresponding values of A_{TT} in (9.8).⁵⁶⁾ Moreover, the asymmetries found at J-PARC are in general larger than those at RHIC energies, because the parton distributions for larger $x_{1,2}^0 \gtrsim 0.2$ are probed, where the enhancement mechanism of the denominator of (9.4) by the unpolarized sea distributions, as noted in Figs. 4(a) and (b), is absent.*)

Finally, we emphasize that in all cases considered in Figs. 3-5, we have observed that $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ in (9.4) in the flat region is considerably larger than the corresponding A_{TT} in (9.8). Indeed, this is generally true for the tDY in pp collisions, regardless of the specific kinematics or the detailed behavior of nonperturbative inputs, because this phenomenon is mainly governed by the partonic mechanism associated with the soft gluon resummation at the NLL level, as demonstrated in Fig. 2. By contrast, apparently the absolute magnitudes of both $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ and A_{TT} are influenced by the detailed behavior of the input parton distributions, in particular, at RHIC energies, by their small- x behavior.

9.3. The saddle-point formula incorporating nonperturbative smearing

In the numerical results presented in §9.2, we have observed the universal flat behavior of $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ at small Q_T , where the bulk of the DY pairs is produced. We have also observed that in this flat region, the numerator and denominator of (9.4) are dominated by the NLL resummed components, $\Delta_T \tilde{X}^{\text{NLL}}$ and \tilde{X}^{NLL} , respectively, and as a result, $\mathcal{A}_{TT}^{\text{NLL+LO}}(Q_T)$ is well approximated by $\mathcal{A}_{TT}^{\text{NLL}}(Q_T)$. This suggests that the experimental value of the double

*) It is expected that even larger asymmetries will be observed in future $p\bar{p}$ experiments at GSI.^{67), 68)} The application of our Q_T resummation formalism to $p\bar{p}$ collisions will be presented elsewhere.⁶⁹⁾

transverse-spin asymmetry obtained from the data in the flat region can be compared with the theoretical value of (9.5) directly. Noting that the flat behavior of the NLL asymmetry (9.5) can be extrapolated to $Q_T \rightarrow 0$, as demonstrated in Figs. 3 and 5(a), we can use the relations

$$\mathcal{A}_{TT}(Q_T) \approx \mathcal{A}_{TT}^{\text{NLL}}(Q_T) \approx \mathcal{A}_{TT}^{\text{NLL}}(0) = \frac{1}{2} \cos(2\phi) \frac{\Delta_T \tilde{X}^{\text{NLL}}(0, Q^2, y)}{\tilde{X}^{\text{NLL}}(0, Q^2, y)}, \quad (9.9)$$

for Q_T in the flat region, where $\mathcal{A}_{TT}(Q_T)$ denotes the experimental value of the double transverse-spin asymmetry. In §8, the $Q_T \rightarrow 0$ limit of the NLL resummed components, $\Delta_T \tilde{X}^{\text{NLL}}(0, Q^2, y)$ and $\tilde{X}^{\text{NLL}}(0, Q^2, y)$, has been evaluated analytically using the saddle-point method for the case without the replacement (9.3). In view of (9.9), it would be interesting to extend the results in §8, incorporating the nonperturbative smearing factor as in (9.3).⁵⁶⁾

Starting from (7.24) with the replacement (9.3), we can use logic similar to that used in §8 in order to obtain (8.1), which now reads, at $Q_T = 0$,

$$\tilde{I}_{N_1, N_2}(0, Q^2) = \frac{1}{2} \int_0^{b_c} db b \exp \left[\frac{h^{(0)}(\lambda)}{\alpha_s(Q^2)} + h^{(1)}(\lambda) + R_{N_1}(\lambda) + R_{N_2}(\lambda) - g_{NP} b^2 \right]. \quad (9.10)$$

Because $b^2 = (b_0^2/Q^2) e^{\lambda/(\beta_0 \alpha_s(Q^2))}$ using (5.21), the nonperturbative function produces a “double exponential” behavior, and the saddle-point evaluation with such an integrand is a nontrivial task. Nevertheless, we can check numerically that the integrand of (9.10) has a nice saddle point well below b_c (above 0) for the kinematics of interest. When $g_{NP} \rightarrow 0$, the integrand is controlled by the perturbative functions in (9.10), which was also the case in §8. Contrastingly, when $g_{NP} \rightarrow \infty$, the integrand is controlled solely by the nonperturbative function, such that the integral of (9.10) is dominated by the $b \approx 0$ region. Our task is to find a formula for finite g_{NP} which interpolates between these two extreme cases, and, for this purpose, we perform the saddle-point evaluation of (9.10), assuming $-g_{NP} b^2 = -g_{NP} (b_0^2/Q^2) e^{\lambda/(\beta_0 \alpha_s(Q^2))}$ as a leading term in the exponent. As a result, we obtain (8.2), where $\zeta^{(0)}(\lambda)$ is now replaced as

$$\zeta^{(0)}(\lambda) \rightarrow -\frac{\lambda}{\beta_0 \alpha_s(Q^2)} - \frac{h^{(0)}(\lambda)}{\alpha_s(Q^2)} + \frac{g_{NP} b_0^2}{Q^2} e^{\frac{\lambda}{\beta_0 \alpha_s(Q^2)}}, \quad (9.11)$$

and correspondingly the equation (8.5) to determine the saddle point is replaced by

$$1 - \frac{A_q^{(1)}}{2\pi\beta_0} \frac{\lambda_{SP}}{1 - \lambda_{SP}} = \frac{g_{NP} b_0^2}{Q^2} e^{\frac{\lambda_{SP}}{\beta_0 \alpha_s(Q^2)}}. \quad (9.12)$$

An important observation is that the ratio $[h^{(1)}(\lambda) + R_{N_1}(\lambda) + R_{N_2}(\lambda)]/\zeta^{(0)}(\lambda)$ with (9.11) actually behaves as a quantity of order $\alpha_s(Q^2)$ in the relevant region, $\lambda \approx \lambda_{SP}$, of the

integration in (8·2), even for nonzero $g_{NP} \simeq 0.5 \text{ GeV}^2$. This fact supports the validity of the present approach, incorporating the nonperturbative function as (9·11). Also, this fact suggests that we may use an order counting that is similar to that explained below (8·7) for the contributions arising in the saddle-point evaluation. This is apparently correct when g_{NP} is sufficiently small, but in general it has to be checked. Here we assume the same order counting as in §8, and check the accuracy of the resulting formula below, comparing it with the result of the numerical b -integration of the original formula, (7·24) with (9·3) (see Table I below).

Now we can perform the saddle-point evaluation of (8·2) at NLL accuracy and also a similar calculation starting from (7·38) for the unpolarized case. Immediately we find that the results (8·8)-(8·12) with (9·11) and (9·12) hold, up to the NNLL corrections corresponding to the $\mathcal{O}(\alpha_s)$ effects.⁵⁶⁾ These results realize the “degree 0” approximation of §6.3. Also, it is straightforward to show that these results reproduce the results in the two extreme limits $g_{NP} \rightarrow 0$ and $g_{NP} \rightarrow \infty$, as desired for the “interpolation formula” mentioned below (9·10). Taking the ratio of the saddle point formulae (8·10) and (8·12), we obtain,⁵⁶⁾ for the RHS of (9·9),

$$\mathcal{A}_{TT}^{\text{NLL}}(0) = \frac{1}{2} \cos(2\phi) \frac{\Delta_T \tilde{X}^{\text{NLL}}(0, Q^2, y)}{\tilde{X}^{\text{NLL}}(0, Q^2, y)} = \frac{1}{2} \cos(2\phi) \frac{\delta H(x_1^0, x_2^0; b_0^2/b_{SP}^2)}{H(x_1^0, x_2^0; b_0^2/b_{SP}^2)}, \quad (9·13)$$

up to NLL accuracy. Here, the common factor in (8·10) and (8·12), associated with the “very large perturbative effects” due to the Sudakov factor, as well as with the Gaussian smearing factor involving g_{NP} , has canceled out.

The new scale, b_0/b_{SP} , must be determined by solving (9·12) numerically, substituting $A_q^{(1)} = 2C_F$ from (5·13) and the input values for Q and g_{NP} , but it is useful to consider its general behavior: The LHS of (9·12) equals 1 at $\lambda_{SP} = 0$, decreases as a concave function as λ_{SP} increases, and vanishes at $\lambda_{SP} = 1/[1 + A_q^{(1)}/(2\pi\beta_0)] \cong 0.6$, which corresponds to the solution (8·6) in the $g_{NP} = 0$ case, while the RHS is in general much smaller than 1 at $\lambda_{SP} = 0$, increases as a convex function as λ_{SP} increases, and is larger than 1 at $\lambda_{SP} \simeq 1$. Thus, the solution of (9·12) corresponds to the case with $\text{LHS} = \text{RHS} \simeq 1/2$, more or less independently of the specific values of Q and g_{NP} , so that we get $b_0/b_{SP} \simeq b_0\sqrt{2g_{NP}}$. This result depends only weakly on the nonperturbative parameter g_{NP} , and it suggests that one may always use $b_0/b_{SP} \simeq 1 \text{ GeV}$ for the cases of our interest, in which Q is on the order of several GeV and $g_{NP} \simeq 0.5 \text{ GeV}^2$, as in Figs. 2-5. The actual numerical solution of (9·12) verifies these simple considerations at the level of 20% accuracy. This fact is particularly helpful in the first attempt to compare (9·13) with the experimental data using (9·9), so as to extract the transversity distributions.

The above considerations also demonstrate that (9·10) is dominated by the region of b near $b_{SP} \simeq 1 \text{ GeV} \sim 1/\Lambda_{\text{QCD}}$, corresponding to the boundary of perturbative and nonperturbative physics. We can “safely” treat such a long-distance region, owing to the fact that the nonperturbative smearing (9·3) suppresses the too long-distance region, $b \gg 1/\Lambda_{\text{QCD}}$ [see the discussion below (8·14)]. Moreover, the model dependence due to the specific choice of g_{NP} largely cancels in the final result (9·13).

Table I. The $Q_T \rightarrow 0$ limit of $\mathcal{A}_{TT}^{\text{NLL}}(Q_T)$ in (9·5) for RHIC and J-PARC kinematics. NB is obtained using the numerical b -integration of (7·23), (7·24) and (7·38) with (9·3) and $g_{NP} = 0.5 \text{ GeV}^2$; SP-I and SP-II are the results of the saddle-point formula (9·13) with (9·12) using the evolution operators from Q to b_0/b_{SP} , to NLL accuracy and to the customary NLO accuracy, respectively.

	$\sqrt{S} = 200 \text{ GeV}, \quad y = 2$					$\sqrt{S} = 10 \text{ GeV}, \quad y = 0$		
Q	2GeV	5GeV	8GeV	15GeV	20GeV	2GeV	2.5GeV	3.5GeV
NB	3.8%	4.9%	6.1%	8.2%	9.4%	13.4%	14.0%	14.9%
SP-I	4.3%	5.4%	6.6%	8.7%	9.8%	14.1%	14.5%	14.8%
SP-II	7.3%	8.7%	9.8%	11.8%	12.7%	14.7%	14.8%	14.2%

In Table I, “NB” lists $\mathcal{A}_{TT}^{\text{NLL}}(Q_T = 0)$ obtained from (9·5) using the numerical b -integration of (7·23), (7·24) and (7·38), with the kinematics of Fig. 4(a) for RHIC and of Fig. 5(b) for J-PARC. The result for $Q = 5 \text{ GeV}$ at RHIC and that for $Q = 2 \text{ GeV}$ at J-PARC coincide with the $Q_T \rightarrow 0$ limit of the dot-dashed curve in Figs. 3 and 5(a), respectively. The row labeled “SP-I” lists the results obtained using the saddle-point formula (9·13): We use b_0/b_{SP} obtained as the solution of (9·12) with $g_{NP} = 0.5 \text{ GeV}^2$, and, according to (8·8), (8·9) and similar formulae for the unpolarized case, the parton distributions participating in (9·13) are obtained by evolving the customary NLO distributions at the scale Q to b_0/b_{SP} with the NLO evolution operators up to NLL accuracy, like (7·18). Here the “customary NLO distributions” are constructed as described above (9·3). Comparing NB and SP-I, we observe the remarkable accuracy of our simple analytic formula (9·13) for both RHIC and J-PARC, reproducing the results of NB to 10% accuracy, i.e., to the canonical size of $\mathcal{O}(\alpha_s)$ corrections associated with the NLL accuracy.^{*)} This supports the assumption concerning the order counting in our saddle-point evaluation, discussed below (9·12).

Although the compact formula (9·13) is reminiscent of $\mathcal{A}_{TT}^{\text{LL}}(Q_T)$ in (9·7), which retains only the LL level resummation, or the Q_T -independent asymmetry of (9·8), it is different in

^{*)} If we use the fixed value $b_0/b_{SP} = 1 \text{ GeV}$ for all cases, instead of the solution of (9·12), the results in SP-I change by at most 5% for both RHIC and J-PARC kinematics.

the scale of the parton distributions from those leading-order results, and the only effect of this new scale b_0/b_{SP} allows SP-I to reproduce NB in Table I. Combined with this fact, the above derivation of (9-13), using (8-10) and (8-12), clearly demonstrates the characteristic features of the NLL soft gluon resummation effects on the asymmetries, which lead to the mechanism to “enhance” the dot-dashed curve in Figs. 3 and 5(a): In the $Q_T \rightarrow 0$ limit, the all-order soft-gluon-resummation effects on the asymmetry largely cancel between the numerator and the denominator of (9-13), but certain contributions at the NLL level survive the cancellation and are entirely absorbed into the unconventional scale $b_0/b_{SP} \simeq 1$ GeV for the relevant distribution functions. Combined with the properties of $\mathcal{A}_{TT}^{\text{NLL}}(Q_T)$ in (9-9), this explains why $\mathcal{A}_{TT}^{\text{NLL}}(Q_T)$ is always larger than $\mathcal{A}_{TT}^{\text{LL}}(Q_T)$ in (9-7) or A_{TT} in (9-8) as the NLL-level effects associated with the evolution [compare with the discussion below (9-7)].

Using (9-9), the saddle-point formula (9-13) can be directly compared with the experimental value of the asymmetries $\mathcal{A}_{TT}(Q_T)$, observed around the peak of the Q_T spectrum of the corresponding DY cross sections. This approach is particularly useful, because it does not require as large a numerical computations as that in §§9.1, 9.2. Furthermore, the approach with (9-13) requires one point of caution about the accuracy, associated with the mismatch in classifying the terms between NLL and NLO. We recall that the parton distributions appearing in (9-13) are the NLO distributions up to the corrections at the NNLL level [see (8-8), (8-9)]. To obtain SP-I in Table I, the transversity distributions appearing in the numerator of (9-13) were obtained by evolving the customary NLO transversity $\delta q(x, Q^2)$ at the scale Q , to the scale b_0/b_{SP} using (7-18), which consists of *the NLO evolution operators up to the NNLL corrections*. Therefore, the formula (9-13) can be safely used in the region where the NNLL corrections are small; the NNLL corrections at $Q_T \approx 0$ correspond to $\mathcal{O}(\alpha_s(Q^2))$ effects, applying the order counting of §§6.3 and 8, and should be negligible in general. However, such a straightforward estimate might fail in the edge region of the phase space, as in the small x region. This follows from the fact that the relevant evolution operators (7-18) actually coincide with the leading contributions in the large-logarithmic expansion of the usual LO DGLAP evolution,^{*)} (9-13) would not be accurate when the NLO corrections in the usual DGLAP evolution are large compared with the contributions of (7-18). Such a situation would typically occur in a region with small $x_{1,2}^0$, corresponding to the case with large \sqrt{S} . In Table I, we show other saddle-point results, “SP-II”, using the customary NLO distributions at the scale b_0/b_{SP} for the parton distributions participating in (9-13). Thus, SP-I and SP-II differ by contributions at the NNLL level. The results for SP-II indicate that

^{*)} This fact also suggests that one may use the fixed value $b_0/b_{SP} \simeq 1$ GeV in (9-13) for all Q (and g_{NP}) rather than solving (9-12) numerically for each different input value of Q and g_{NP} , because the sensitivity of the LO evolution with respect to a small change of the scale is weak.

the NNLL corrections are moderate for large \sqrt{S} at RHIC, while they are expected to be small for small \sqrt{S} at J-PARC.

We propose that our simple formula (9.13) is applicable to analysis of low-energy experiments at J-PARC in order to extract the NLO transversity distributions directly from the data. On the other hand, (9.13) will not be very accurate for analyzing the data obtained from RHIC, but it will still be useful for obtaining the first estimate of the transversities. We emphasize that such a (moderate) uncertainty in applying our formula (9.13) to the RHIC case is not caused by the saddle-point evaluation, nor by considering the $Q_T \rightarrow 0$ limit, but rather is inherent in the general Q_T resummation framework which, at the NLL level, implies the use of the evolution operators (7.18) with the LO anomalous dimension. A more accurate treatment of the small- x region of the parton distributions relevant to the RHIC case would require the resummation formula to NNLL accuracy, where the NLO anomalous dimensions participate in the evolution operators (7.18) from Q to b_0/b (see e.g. Ref. 35)).

§10. Conclusions

The era of “testing QCD” is clearly finished and today the improvement of the precision of predictions is one of the most important subjects in QCD. We have reported our analysis of the transversely polarized DY process, which can be measured in ongoing experiments at RHIC and in future experiments at J-PARC, GSI, etc. We have demonstrated that the QCD corrections relevant for the tDY at a measured Q_T are now under control over the entire range of Q_T , so that the dilepton Q_T spectrum as well as the Q_T dependence of the double-spin asymmetry for the tDY is predictable in a systematic QCD factorization framework. In particular, in the small Q_T region, where most of the dileptons are produced in experiments, these observables are dominated by universal logarithmically-enhanced contributions, which we have taken into account by an all-order resummation of multiple soft-gluon emission. Based on this framework, we have performed a detailed numerical study of the observables in the tDY at RHIC and J-PARC, using a model of the transversity which is constructed to satisfy a known theoretical constraint. We find that the logarithmically-enhanced higher-order QCD corrections associated with soft gluon resummation drive a mechanism that amplifies the double transverse-spin asymmetries in the small Q_T region. Quantitatively, our results indicate that the double transverse-spin asymmetries seem to be small at RHIC, while they will be large at J-PARC. This would imply that the measurement of the tDY double transverse-spin asymmetries at RHIC might require considerable effort. Still, we expect the data from RHIC, as well as that from J-PARC, GSI, etc., that are comparable with our predictions. In particular, we have proposed a remarkably simple formula for

the double transverse-spin asymmetries, which provides us with a new direct approach to extracting the transversity distributions from experimental data. We believe that the results presented in this paper will be useful not only for experiments at RHIC and J-PARC but also for future precision measurements.

We have obtained all our results concerning the QCD corrections for the DY in the $\overline{\text{MS}}$ scheme, for the first time, directly in the dimensional regularization with transverse polarization, and our calculation technique can be straightforwardly applied to other polarized and unpolarized processes. As described in this paper, the formalism of transverse-momentum resummation we employed is the same for many other polarized and unpolarized processes, and has a wide range of application, in particular, to the processes observed at LHC, where rigorous treatments of multi-scale problems in QCD are mandatory and require resumming ubiquitous soft gluons. QCD is now facing a new challenging era regarding its precision and applicability. We hope that various kinds of new experiments and theoretical investigations will be able to clarify not only perturbative and nonperturbative aspects of QCD but also the full structure of all interactions in Nature.

Acknowledgments

We thank Hirotaka Shimizu for collaboration in the early stage of this work and Werner Vogelsang, Hiroshi Yokoya and Stefano Catani for valuable discussions and comments. We also express our thanks to Masao Ninomiya for his encouragement to complete the paper. We are sincerely grateful to Mieko Kodaira for her constant support and encouragement during more than ten years of our collaboration. This work was supported by the Grant-in-Aid for Scientific Research Nos. C-16540255, C-16540266 and B-19340063.

Appendix A

— The Transversity Distribution —

The transversity distribution of a quark inside a spin-1/2 hadron h is defined in terms of a matrix element of a non-local light-cone operator as¹⁴⁾

$$\delta q_h(x, \mu^2) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle h(P, S) | \bar{\psi}(0) [0, \lambda n] \not{n} \gamma_5 \not{S} \psi(\lambda n) | h(P, S) \rangle, \quad (\text{A} \cdot 1)$$

where P and S are the momentum and transverse spin of the hadron h , respectively, and n^μ is a light-like vector which satisfies $n^2 = 1$ and $P \cdot n = 1$; for example, when we ignore the nucleon mass, setting $P^2 = 0$, and work in a frame where $P^\mu = (P^+/\sqrt{2})(1, \mathbf{0}, 1)$, we have

$S^\mu = (0, \mathbf{S}, 0)$ and $n^\mu = (1/\sqrt{2}P^+)(1, \mathbf{0}, -1)$. The path-ordered link operator (Wilson line)

$$[0, \lambda n] = \mathcal{P} \exp \left(-ig \int_0^\lambda d\lambda' n \cdot A(\lambda' n) \right), \quad (\text{A}\cdot 2)$$

which connects the two quark operators $\bar{\psi}(0)$ and $\psi(\lambda n)$, is required to ensure the gauge invariance of the non-local operator. In the definition (A.1), the scale μ^2 of the distribution corresponds to the renormalization scale of the non-local operator on the RHS.

The “quark transversity-distribution inside a quark” can be obtained by evaluating the matrix element (A.1), with $h = \text{quark } q'$, in perturbation theory with an appropriate regularization of the IR divergence. Employing the dimensional regularization in $D = 4 - 2\epsilon$ dimensions for the IR and UV divergences and evaluating the matrix element up to one-loop accuracy, (A.1) for $h = q'$ yields^{*)}

$$\delta q_{q'}(x, \mu^2) = \delta_{qq'} \left[\delta(1-x) - \frac{\alpha_s(\mu^2)}{2\pi} \frac{1}{\hat{\epsilon}} \Delta_T P_{qq}(x) \right], \quad (\text{A}\cdot 3)$$

in the $\overline{\text{MS}}$ scheme for renormalization of the non-local operator of (A.1), where $1/\hat{\epsilon} \equiv 1/\epsilon - \gamma_E + \ln(4\pi)$, and $\Delta_T P_{qq}(x)$ is the LO splitting function for the transversity:

$$\Delta_T P_{qq}(x) = C_F \left\{ \frac{2x}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right\}. \quad (\text{A}\cdot 4)$$

We note that we can derive the DGLAP evolution equation at the LO level by differentiating (A.3) with respect to μ^2 . The result is [compare with (5.15)]

$$\mu^2 \frac{\partial}{\partial \mu^2} \delta q_h(x, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dz}{z} \Delta_T P_{qq} \left(\frac{x}{z} \right) \delta q_h(z, \mu^2). \quad (\text{A}\cdot 5)$$

Together with the density distributions $(q_h(x, \mu^2), g_h(x, \mu^2))$ and the helicity distributions $(\Delta q_h(x, \mu^2), \Delta g_h(x, \mu^2))$, the transversity distribution $\delta q_h(x, \mu^2)$ forms a complete set of twist-2 parton distributions for a spin-1/2 hadron h . Here the quark distributions $q_h(x, \mu^2)$ and $\Delta q_h(x, \mu^2)$ are defined by the matrix element (A.1) for the unpolarized and longitudinally-polarized hadron states, with the chiral-odd Dirac structure $\not{n}\gamma_5\not{n}$ replaced by the corresponding chiral-even structures \not{n} and $\not{n}\gamma_5$, respectively. Thus, unlike these two chiral-even quark distributions, the transversity is chiral-odd. Because all gluon distributions at twist-2 are chiral-even, and the QCD interaction conserves chirality, the gluon distributions do not mix into the DGLAP evolution (A.5) for the transversity, even at NLO or higher-order level. As a result, the QCD evolution of the transversity beyond the LO is of the non-singlet type; i.e., it involves only the mixing between the quark and antiquark

^{*)} A similar calculation for the unpolarized quark distribution $q_{q'}(x, \mu^2)$ is described in Ref. 1).

distributions associated with the same flavor, and it is much simpler than the evolution for the other two quark distributions. For example, in the Mellin N -space, the NLO evolution of the transversity distribution is written as

$$\delta q_N^\pm(Q^2) = \left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right]^{-\gamma_{qq,N}^{(0)}/(2\pi\beta_0)} \times \left[1 + \frac{\alpha_s(Q_0^2) - \alpha_s(Q^2)}{4\pi^2\beta_0} \left(\gamma_{qq,N}^{(1)\pm} - \frac{2\pi\beta_1}{\beta_0} \gamma_{qq,N}^{(0)} \right) \right] \delta q_N^\pm(Q_0^2) , \quad (\text{A}\cdot 6)$$

where $\delta q_N^\pm(\mu^2) \equiv \int_0^1 dx x^{N-1} \delta q^\pm(x, \mu^2)$ with $\delta q^\pm(x, \mu^2) \equiv \delta q_h(x, \mu^2) \pm \delta \bar{q}_h(x, \mu^2)$. The LO anomalous dimension is given by⁷⁰⁾

$$\gamma_{qq,N}^{(0)} = \int_0^1 dx x^{N-1} \Delta_T P_{qq}(x) = C_F \left[-2S_1(N) + \frac{3}{2} \right] , \quad (\text{A}\cdot 7)$$

and the NLO anomalous dimensions in the $\overline{\text{MS}}$ scheme are^{19), 71)}

$$\begin{aligned} \gamma_{qq,N}^{(1)\eta} = & C_F^2 \left[\frac{3}{8} + \frac{1-\eta}{N(N+1)} - 3S_2(N) - 4S_1(N) \left(S_2(N) - S'_2\left(\frac{N}{2}\right) \right) \right. \\ & \left. - 8\tilde{S}(N) + S'_3\left(\frac{N}{2}\right) \right] \\ & + \frac{1}{2} C_F N_c \left[\frac{17}{12} - \frac{1-\eta}{N(N+1)} - \frac{134}{9} S_1(N) + \frac{22}{3} S_2(N) \right. \\ & \left. + 4S_1(N) \left(2S_2(N) - S'_2\left(\frac{N}{2}\right) \right) + 8\tilde{S}(N) - S'_3\left(\frac{N}{2}\right) \right] \\ & + \frac{2}{3} C_F N_f T_R \left[-\frac{1}{4} + \frac{10}{3} S_1(N) - 2S_2(N) \right] , \end{aligned} \quad (\text{A}\cdot 8)$$

where $\eta = \pm$, and the harmonic sums are defined by

$$S_k(N) = \sum_{j=1}^N \frac{1}{j^k} , \quad (\text{A}\cdot 9)$$

$$S'_k\left(\frac{N}{2}\right) = 2^{k-1} \sum_{j=1}^N \frac{1 + (-1)^j}{j^k} , \quad (\text{A}\cdot 10)$$

$$\tilde{S}(N) = \sum_{j=1}^N \frac{(-1)^j}{j^2} S_1(j) . \quad (\text{A}\cdot 11)$$

The first moment of the LO splitting function is negative ($\gamma_{qq,N=1}^{(0)} = -\frac{2}{3}$), so that the transverse polarization in total (the “tensor charge”) decreases as the scale increases.*)

*) A pedagogical explanation of the transverse-spin flip by collinear radiation is given in the appendix of Ref. 72).

We note that the dominant large- z behavior (5.17) of the DGLAP splitting functions (5.16) reflects the dominant large N behavior of the corresponding anomalous dimensions (7.13). For the present case, the anomalous dimensions given in (A.7) and (A.8) at large N behave as

$$\gamma_{qq,N}^{(0)} = -A_q^{(1)} (\ln N + \gamma_E) - \frac{B_q^{(1)}}{2} + \cdots, \quad (\text{A.12})$$

where the ellipses stand for the terms that vanish as $N \rightarrow \infty$, and

$$\gamma_{qq,N}^{(1)\eta} = -A_q^{(2)} (\ln N + \gamma_E) + \cdots, \quad (\text{A.13})$$

where the ellipses stand for the corrections of $\mathcal{O}(1)$. The coefficients on the RHS of (A.12) and (A.13) coincide with (5.13). The same asymptotic behavior arises in the anomalous dimensions for the density and helicity distributions $q_h(x, \mu^2)$ and $\Delta q_h(x, \mu^2)$ (see e.g. Ref. 73)).

Appendix B

— The Cross Section Integrated over the Rapidity y —

Integrating (4.29) over the rapidity y of the dilepton in the final state, we obtain the rapidity-integrated cross section in the $\overline{\text{MS}}$ factorization scheme,

$$\frac{\Delta_T d\sigma}{dQ^2 dQ_T^2 d\phi_{k_1}} = \mathcal{N} \cos(2\phi_{k_1} - \phi_1 - \phi_2) \int dy [\Delta_T X(Q_T^2, Q^2, y) + \Delta_T Y(Q_T^2, Q^2, y)], \quad (\text{B.1})$$

including the $\mathcal{O}(\alpha_s)$ QCD mechanism. We use the following integration formulae:

$$\int dy \delta H(x_1^0, x_2^0) = \int dx_1 dx_2 \delta H(x_1, x_2) \delta(x_1 x_2 - \tau), \quad (\text{B.2})$$

$$\int dy \int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) \delta H\left(\frac{x_1^0}{z}, x_2^0\right) = \int \frac{dx_1 dx_2}{x_1 x_2} \Delta_T P_{qq}(\tau_{12}) \delta H(x_1, x_2) \theta(x_1 x_2 - \tau), \quad (\text{B.3})$$

$$\int dy \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{x_1 - x_1^+} \frac{\tau \delta H(x_1, x_2^*)}{x_1 x_2^*} = \int \frac{dx_1 dx_2}{x_1 x_2} \frac{\tau_{12} \delta H(x_1, x_2)}{\sqrt{(1 - \tau_{12})^2 - 4\rho_{12}}} \theta(x_1 x_2 - \tau_+), \quad (\text{B.4})$$

and

$$\begin{aligned} & \int dy \left(\int dx_1 \delta H_1 + \delta H(x_1^0, x_2^0) \ln \frac{1 - x_1^+}{1 - x_1^0} \right) \\ &= \int_{\tau_+}^1 \frac{dx_1}{x_1} \delta H(x_1, \tau/x_1) \ln \frac{\sqrt{(x_1 - \tau_+)(x_1 - \tau_-)} + x_1 - \tau - 2\rho}{2\sqrt{\rho(\tau + \rho)}} \\ & \quad - \int_{\tau}^1 \frac{dx_1}{x_1} \delta H(x_1, \tau/x_1) \ln \frac{x_1 - \tau}{\sqrt{\rho\tau}} \end{aligned}$$

$$\begin{aligned}
& + \int \frac{dx_1 dx_2}{x_1 x_2} \frac{\tau_{12} \delta H(x_1, x_2) - \delta H(x_1, \tau/x_1)}{\sqrt{(1 - \tau_{12})^2 - 4\rho_{12}}} \theta(x_1 x_2 - \tau_+) \\
& - \int \frac{dx_1 dx_2}{x_1 x_2} \frac{\tau_{12} \delta H(x_1, x_2) - \delta H(x_1, \tau/x_1)}{1 - \tau_{12}} \theta(x_1 x_2 - \tau),
\end{aligned} \tag{B.5}$$

where $\delta H(x_1, x_2)$ denotes the product of the transversity distributions, (2.4), suppressing the scale dependence, δH_1 is defined in (4.25), τ is given in (4.7), and we have

$$\rho = \frac{Q_T^2}{S}, \quad \sqrt{\tau_{\pm}} = \sqrt{\tau + \rho} \pm \sqrt{\rho}, \quad \tau_{12} = \frac{\tau}{x_1 x_2}, \quad \rho_{12} = \frac{\rho}{x_1 x_2}. \tag{B.6}$$

We thus get

$$\begin{aligned}
\int dy \Delta_T X(Q_T^2, Q^2, y) &= \int \frac{dx_1}{x_1} \delta H(x_1, \tau/x_1; \mu_F^2) \\
&\times \left\{ \delta(Q_T^2) + \frac{\alpha_s}{2\pi} C_F \left[(\pi^2 - 8) \delta(Q_T^2) + 2 \left(\frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ - \frac{3}{(Q_T^2)_+} \right] \right\} \\
&+ \frac{\alpha_s}{\pi} \left(\frac{1}{(Q_T)_+} + \delta(Q_T^2) \ln \frac{Q^2}{\mu_F^2} \right) \int_{\tau}^1 \frac{dx_1}{x_1} \int_{\tau/x_1}^1 \frac{dx_2}{x_2} \Delta_T P_{qq}(\tau_{12}) \delta H(x_1, x_2; \mu_F^2)
\end{aligned} \tag{B.7}$$

and

$$\begin{aligned}
& \int dy \Delta_T Y(Q_T^2, Q^2, y) \\
&= \frac{\alpha_s}{\pi} C_F \left\{ \left[\frac{4}{Q^2} - \frac{6}{Q_T^2} \ln \left(1 + \frac{Q_T^2}{Q^2} \right) \right] \int_{\tau_+}^1 \frac{dx_1}{x_1} \int_{\tau_+/x_1}^1 \frac{dx_2}{x_2} \frac{\tau_{12} \delta H(x_1, x_2; \mu_F^2)}{\sqrt{(1 - \tau_{12})^2 - 4\rho_{12}}} \right. \\
&+ \frac{2}{Q_T^2} \left[\int_{\tau_+}^1 \frac{dx_1}{x_1} \int_{\tau_+/x_1}^1 \frac{dx_2}{x_2} \frac{\tau_{12} \delta H(x_1, x_2; \mu_F^2) - \delta H(x_1, \tau/x_1; \mu_F^2)}{\sqrt{(1 - \tau_{12})^2 - 4\rho_{12}}} \right. \\
&\quad \left. \left. - \int_{\tau}^1 \frac{dx_1}{x_1} \int_{\tau/x_1}^1 \frac{dx_2}{x_2} \frac{\tau_{12} \delta H(x_1, x_2; \mu_F^2) - \delta H(x_1, \tau/x_1; \mu_F^2)}{1 - \tau_{12}} \right] \right. \\
&+ \int_{\tau_+}^1 \frac{dx_1}{x_1} \delta H(x_1, \tau/x_1; \mu_F^2) \ln \frac{x_1 - \tau - 2\rho + \sqrt{(x_1 - \tau_+)(x_1 - \tau_-)}}{2\sqrt{\rho(\tau + \rho)}} \\
&\quad \left. \left. - \int_{\tau}^1 \frac{dx_1}{x_1} \delta H(x_1, \tau/x_1; \mu_F^2) \ln \frac{x_1 - \tau}{\sqrt{\rho\tau}} \right\}.
\end{aligned} \tag{B.8}$$

For $Q_T^2 > 0$, the delta function $\delta(Q_T^2)$ involved in (B.7) vanishes, and, after some calculation, (B.1) with (B.7) and (B.8) reduces to

$$\begin{aligned}
\frac{\Delta_T d\sigma}{dQ^2 dQ_T^2 d\phi_{k_1}} &= \mathcal{N} \cos(2\phi_{k_1} - \phi_1 - \phi_2) \frac{\alpha_s}{\pi} C_F \left[\frac{2}{Q_T^2} + \frac{4}{Q^2} - \frac{6}{Q_T^2} \ln \left(1 + \frac{Q_T^2}{Q^2} \right) \right] \\
&\times \int_{\tau_+}^1 \frac{dx_1}{x_1} \int_{\tau_+/x_1}^1 \frac{dx_2}{x_2} \frac{\tau_{12} \delta H(x_1, x_2; \mu_F^2)}{\sqrt{(1 - \tau_{12})^2 - 4\rho_{12}}}.
\end{aligned} \tag{B.9}$$

This coincides with the result obtained by imposing the restriction $Q_T^2 > 0$ in Ref. 16), and it provides the LO QCD prediction in the large Q_T region.

To obtain the QCD prediction for the rapidity-integrated tDY cross section over the entire region of Q_T , we integrate (7·25) over y . This gives the rapidity-integrated NLL+LO cross section in the $\overline{\text{MS}}$ scheme. The y -integral of $\Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y)$, as well as $\Delta_T \tilde{Y}(Q_T^2, Q^2, y)$, can be performed straightforwardly using (B·2)-(B·5), similarly to the above fixed-order case. For $\Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y)$, the result can be expressed in the Mellin moment space with respect to $\tau = x_1^0 x_2^0$ at fixed Q : Changing the integration variables from x_1^0 and x_2^0 to τ and y in the definition of the double Mellin moment (7·23) [see (7·6)], we obtain $\int dx_1^0 dx_2^0 (x_1^0)^{N_1-1} (x_2^0)^{N_2-1} \Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) = \int d\tau dy \tau^{(N_1+N_2)/2-1} e^{(N_1-N_2)y} \Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y)$. Setting $N_1 = N_2$ in this relation, we get the Mellin-space representation of the corresponding y -integral as

$$\begin{aligned} \int_0^1 d\tau \tau^{N_1-1} \left[\int dy \Delta_T \tilde{X}^{\text{NLL}}(Q_T^2, Q^2, y) \right] \\ = \left[1 + \frac{\alpha_s(Q^2)}{2\pi} C_F(\pi^2 - 8) \right] \delta H_{N_1, N_1}(Q^2) \tilde{I}_{N_1, N_1}(Q_T^2, Q^2), \end{aligned} \quad (\text{B}·10)$$

with $\tilde{I}_{N_1, N_1}(Q_T^2, Q^2)$ of (7·24).

Appendix C

—— Cancellation at $Q_T = 0$ in Eq. (7·26) ——

In this appendix, we show that the terms proportional to the delta function $\delta(Q_T^2)$ cancel out in the difference $\Delta_T X - \Delta_T \tilde{X}^{\text{NLL}}|_{\text{fo}}$ in (7·26), and as a result, $\Delta_T \tilde{Y}$ differs from $\Delta_T Y$ only by the terms that are less singular than $1/Q_T^2$ or $\delta(Q_T^2)$ as $Q_T^2 \rightarrow 0$. This shows that the NLL+LO cross section (7·25) is smooth and well-defined in the entire region of Q_T , including $Q_T = 0$. For this purpose, we explicitly show that [see (7·27)] the relation

$$\lim_{\epsilon \rightarrow 0} \int_0^{\epsilon^2} dQ_T^2 (\mathcal{I}_n - \tilde{\mathcal{I}}_n) = \lim_{\epsilon \rightarrow 0} \left(- \int_{\epsilon^2}^{Q^2} dQ_T \mathcal{I}_n + \int_{\epsilon^2}^{\infty} dQ_T \tilde{\mathcal{I}}_n \right) = 0 \quad (\text{C}·1)$$

holds for $n = 1$ and 2 , which means that the delta function $\delta(Q_T^2)$ implicit in the $+$ distributions in $\mathcal{I}_{1,2}$ cancels with that in $\tilde{\mathcal{I}}_{1,2}$.

The first equality in (C·1) follows from (7·33). To show the second equality, it is convenient to consider the generating functions of \mathcal{I}_n and $\tilde{\mathcal{I}}_n$. The explicit form of the former is already given in (6·4) as

$$\mathcal{I}(\eta) \equiv \sum_{n=0}^{\infty} \frac{\mathcal{I}_n}{n!} \eta^n$$

$$= \left[\delta(Q_T^2) - \sum_{m=0}^{\infty} \frac{\eta^{m+1}}{m!} \left(\frac{\ln^m(Q^2/Q_T^2)}{Q_T^2} \right)_+ \right] \exp \left[-2 \sum_{r=1}^{\infty} \frac{\zeta(2r+1)}{2r+1} \eta^{2r+1} \right], \quad (\text{C}\cdot 2)$$

which can be obtained from the relations

$$\mathcal{I}(\eta) = \frac{1}{2} \int_0^{\infty} db b J_0(b Q_T) \left[\frac{b^2 Q^2}{b_0^2} \right]^{\eta} = \frac{\Gamma(1+\eta)}{\Gamma(-\eta)} \frac{1}{Q_T^2} \left[\frac{4 Q^2}{b_0^2 Q_T^2} \right]^{\eta} \quad (\text{C}\cdot 3)$$

and $\Gamma(1+\eta)/\Gamma(1-\eta) = (b_0^2/4)^{\eta} \exp \left[-2 \sum_{r=1}^{\infty} \frac{\zeta(2r+1)}{2r+1} \eta^{2r+1} \right]$. The Q_T integration of (C·2) reads

$$\int_{\epsilon^2}^{Q^2} dQ_T^2 \mathcal{I}(\eta) = \left\{ 1 - \left(\frac{Q^2}{\epsilon^2} \right)^{\eta} \right\} \exp \left[-2 \sum_{r=1}^{\infty} \frac{\zeta(2r+1)}{2r+1} \eta^{2r+1} \right]. \quad (\text{C}\cdot 4)$$

Next, the generating function of $\tilde{\mathcal{I}}_n$,

$$\tilde{\mathcal{I}}(\eta) \equiv \sum_{n=0}^{\infty} \frac{\tilde{\mathcal{I}}_n}{n!} \eta^n, \quad (\text{C}\cdot 5)$$

is given by³⁵⁾

$$\tilde{\mathcal{I}}(\eta) = \frac{1}{2} \int_0^{\infty} db b J_0(b Q_T) \left[\frac{b^2 Q^2}{b_0^2} + 1 \right]^{\eta} = \frac{b_0^2}{Q^2} \frac{2^{\eta}}{\Gamma(-\eta)} \frac{K_{1+\eta}(b_0 Q_T/Q)}{(b_0 Q_T/Q)^{1+\eta}}, \quad (\text{C}\cdot 6)$$

and the Q_T integration of this function is obtained as

$$\int_{\epsilon^2}^{\infty} dQ_T^2 \tilde{\mathcal{I}}(\eta) = \frac{2^{1+\eta}}{\Gamma(-\eta)} \frac{K_{\eta}(b_0 \epsilon/Q)}{(b_0 \epsilon/Q)^{\eta}}. \quad (\text{C}\cdot 7)$$

Expanding $K_{\eta}(b_0 \epsilon/Q)$ about $\epsilon = 0$, we can derive

$$\int_{\epsilon^2}^{\infty} dQ_T^2 \tilde{\mathcal{I}}(\eta) = \left\{ 1 - \left(\frac{Q^2}{\epsilon^2} \right)^{\eta} \exp \left[-2 \sum_{r=1}^{\infty} \frac{\zeta(2r+1)}{2r+1} \eta^{2r+1} \right] \right\} [1 + \mathcal{O}(\epsilon^2)]. \quad (\text{C}\cdot 8)$$

Taylor expanding (C·4) and (C·8) about $\eta = 0$ and comparing the results, we see that (C·1) is valid for $n = 1$ and 2 . For $n \geq 3$, (C·1) is not valid, because terms including the zeta function $\zeta(n)$ appear differently in (C·4) and (C·8).

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